GOBIERNO DE CHILE
CONICYT

INFORME FINAL

Código Proyecto: D03I1039

Nombre del Proyecto: DESARROLLO DE HERRAMIENTAS COMPUTACIONALES PARA OPTIMIZAR LA GESTION DE CARTERAS DE INVERSION EN MERCADOS EMERGENTES: APLICACION A LOS FONDOS DE PENSIONES EN CHILE

Instituciones Participantes: Pontificia Universidad Católica de Chile

Otros Participantes: AFP Habitat S.A.
Dictuc S.A./ RiskAmerica

Director del Proyecto: .Gonzalo Cortazar Sanz, Firma:... ..

Fecha de emisión : 30/07/2007

GOBIERNO DE CHILE
CONICYT
FONDEF
FOMENTO AL DESARROLLO CIENTIFICO Y TECNOLOGICO

COMISION NACIONAL DE INVESTIGACION CIENTIFICA Y TECNOLOGICA
BERNARDA MORIN 495 • CASILLA 297-V• CORREO 21• FONO: 3654400 • FAX: 6551394 • CHILE

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I PARTE

ACTA DE TERMINO DEL PROYECTO

A.- **IDENTIFICACIÓN DEL PROYECTO**

Nombre del Proyecto: **DESARROLLO DE HERRAMIENTAS COMPUTACIONALES PARA OPTIMIZAR LA GESTION DE CARTERAS DE INVERSION EN MERCADOS EMERGENTES: APLICACION A LOS FONDOS DE PENSIONES EN CHILE**

Código FONDEF del Proyecto: D03I1039

Director del Proyecto: Gonzalo Cortazar Sanz

Instituciones Beneficiarias: Pontificia Universidad Católica de Chile

Empresas participantes: AFP Habitat S.A.
Dictuc S.A./ RiskAmerica

Otras Instituciones participantes:

Montos comprometidos en contrato:	**Fondef**	**$ 158,00**	**millones**
	Instituciones	**$ 122,85**	**millones**
	Empresas	**$ 228,50**	**millones**
	Otros	**$**	**millones**

B.- **EJECUCIÓN**

1. **Fecha toma de razón:** **06/12/2004**

2. **Plazo contractual en meses:** **28 meses**

3. **Fecha efectiva de inicio:** **06/12/2004**

4. **Fecha de término efectiva):** **30/07/2007**

5. **Duración efectiva:** **31 meses**

6.

El proyecto tuvo una duración total de 31 meses. El financiamiento por FONDEF se efectuó durante 28 meses.

Montos efectivamente aportados:	**Fondef**	**$**	**155,303 millones**
	Instituciones	**$**	**122,873 millones**
	Empresas	**$**	**229,913 millones**
	Otros	**$**	**0 millones**

- **Costo Total del Proyecto**

  El costo total del proyecto fue de **509.35** millones de pesos.

- **Aportes de Fondef.**

El monto total rendido y aprobado por FONDEF es de $ 153.632.730 ciento cincuenta y tres millones seiscientos treinta y dos mil setecientos treinta pesos. La diferencia de **$ 1.670.564 un millón seiscientos setenta mil quinientos sesenta y cuatro** pesos con respecto a lo girado por FONDEF ha sido reintegrada mediante cheque nominativo cruzado a nombre de CONICYT por el mismo monto.

Las instituciones beneficiaras declaran haber utilizado el subsidio para financiar los recursos que consulta el proyecto.

- **Aporte de los beneficiarios.**

La institución hizo aportes a la ejecución del proyecto con recursos valorados en **$ 352,786** millones de pesos. Dicho monto lo enteraron con **$ 122,873** millones de pesos en recursos de las propias instituciones beneficiarias y con **$ 229,913** millones de pesos en recursos aportados por las empresas y otras contrapartes del proyecto.

Los recursos declarados de contraparte, satisfacen el porcentaje mínimo exigible por bases del concurso.

Las instituciones beneficiarias declaran que los montos detallados de los aportes de las diferentes fuentes se encuentran en el ANEXO 3 de este informe.

7. Objetivos y Resultados obtenidos

Objetivos Generales

El objetivo principal del proyecto es desarrollar herramientas, aplicaciones y servicios computacionales, que aprovechen en forma efectiva las tecnologías asociadas a internet para modernizar el sistema financiero nacional apoyando una mejor gestión de carteras de inversión en activos transados en el mercado nacional e internacional.

Los desarrollos se focalizarán preferentemente en la problemática de los fondos de pensiones, pero sus resultados impactarán la gestión de otras carteras de inversión como las administradas por compañías de seguros y fondos mutuos, entre otros.

Esta modernización se apoyará tanto en el estado del arte metodológico mundial como en investigación científica que aborde la problemática de mercados financieros poco profundos como el nacional, con activos que se transan con una baja frecuencia (thin markets), lo que dificulta el uso de numerosos procedimientos y metodologías utilizadas en los mercados desarrollados.

De este modo se pretende (1) apoyar una gestión más eficiente de las carteras al incluirse mayor información relativa a retornos y riesgos involucrados, (2) hacer un análisis de estrategias de inversión que apoye la asignación de activos (asset allocation), (3) apoyar funciones de medición y gestión del riesgo y (4) establecer un conjunto de benchmarks para diversas carteras de inversión. Todo lo anterior busca favorecer la gestión e información para directivos y usuarios y, en ultimo término, la competitividad y desempeño de la industria.

Objetivos Específicos

1) Generar conocimiento científico

2) Realizar desarrollos tecnológicos

3) Generar información

4) Desarrollar mecanismos de transferencia

5) Formar investigadores y profesionales especializados

8. Objetivos y Resultados No obtenidos

No hay

9. Apreciación de impacto del proyecto.

Las instituciones declaran que de acuerdo a su evaluación de impacto, el proyecto ha generado y está en proceso de generar los siguientes impactos.

- **Científico-Tecnológico**
  - **Obtenido*:*** Nuevas metodologías de valorización y gestión del riesgo principalmente para mercados con pocas transacciones como son los mercados de economías emergentes como la chilena. Esto se ha traducido en publicaciones, tesis de magíster y presentaciones en conferencias académicas internacionales.
  - **En proceso de obtención:** Nuevas publicaciones en preparación orientadas a formas de gestionar carteras de inversión y a modelos multi-activos.

- **Económico-Social**

    - **Obtenido*:*** Mejor valorización de carteras de inversión para todos los Fondos Mutuos del País y para algunas otras instituciones financieras del país, lo que transparenta los mercados y permite una mejor competencia y asignación de recursos financieros.

    - **En proceso de obtención:** Mejores decisiones de inversión y de gestión del riesgo para carteras de inversión de las instituciones financieras del país a medida que vayan adoptando las herramientas que actualmente están en fase de prueba.

- **Institucional**

    - ***Obtenido:*** -Fortalecimiento del FINlabUC-Laboratorio de Investigación Avanzada en Finanzas tanto en actividad, reconocimiento y equipamiento.
      -Fortalecimiento del programa de Magíster en Ciencias de la Ingeniería con incremento en el número de alumnos que se especializan en finanzas a nivel de postgrado
      -Fortalecimiento de Relaciones con Sector Productivo
      -Fortalecimiento de las relaciones de Cooperación Internacional

    - **En proceso de obtención:** -Incremento en los fortalecimientos institucionales anteriores.

- **Ambiental**

    - **Obtenido:** No existen

    - **En proceso de obtención:** No existen

9. Plan de trabajo.

El Plan de trabajo se estructuró en torno al desarrollo de 5 subtemas que se denominaron: PortfolioValue, PortfolioBenchmarks, PortfolioRisk, RiskMatrix, AssetAllocation, todos los cuales en conjunto reciben la denominación RiskPortfolio.

Estos subtemas se estructuraron como subproyectos de I&D dando origen a Tesis de Magíster y Memorias de Título, Presentaciones a Congresos, Publicaciones, Módulos computacionales y finalmente 3 servicios: SVC, Indices, y Portfolio.

Las instituciones declaran que el plan de trabajo que representa las actividades del proyecto se encuentra en el ANEXO 2 de este informe.

10. Infraestructura y bienes adquiridos por el proyecto

Las instituciones beneficiarias declaran tener inventariados todos los bienes adquiridos por el proyecto y declarados en ANEXO 4 de este Informe, los que están a cargo de personal de la institución y se encuentran asignados a las unidades institucionales que se indican en ese Anexo.

11. **Plan de Continuidad**. Las instituciones se comprometen a:

- **a.-** **La mantención y consolidación de las líneas de investigación asociadas al proyecto, por un plazo no inferior a 3 años**
- **b.-** **El uso de la infraestructura y equipamiento asociado al proyecto en el apoyo a proyectos de I&D o servicios C&T con alto impacto económico social.**
- **c.-** **La valorización, comercialización y transferencia de los resultados del proyecto que se requiera para maximizar los impactos .**
- **d.-** **La protección de los resultados así como el beneficio en términos razonablemente onerosos para la institución a partir de las rentas que de ellos se obtengan.**

INSTITUCION BENEFICIARIA	**INSTITUCION BENEFICIARIA**	**INSTITUCION BENEFICIARIA**
NOMBRE: Pontificia Universidad Católica de Chile	NOMBRE:	NOMBRE:
NOMBRE REPRESENTE LEGAL Carlos Vio Lagos Vicerrector Adjunto de Investigación y Doctorado	NOMBRE REPRESENTE LEGAL	NOMBRE REPRESENTE LEGAL
FIRMA	FIRMA	FIRMA

LAS INSTITUCIONES BENEFICIARIAS DECLARAN ESTAR EN CONOCIMIENTO Y DE ACUERDO CON EL CONTENIDO TOTAL DE ESTE INFORME Y QUE LOS DATOS REGISTRADOS EN ESTA DECLARACIÓN CORRESPONDEN A UN RESUMEN DE LOS DETALLADOS EN ÉL.

II PARTE. INFORME EJECUTIVO

Código Proyecto: D03I1039

Nombre del Proyecto: DESARROLLO DE HERRAMIENTAS COMPUTACIONALES PARA OPTIMIZAR LA GESTION DE CARTERAS DE INVERSION EN MERCADOS EMERGENTES: APLICACION A LOS FONDOS DE PENSIONES EN CHILE

La información entregada en esta parte del documento debe ser sólo la que puede ser de dominio público.

COMISION NACIONAL DE INVESTIGACION CIENTIFICA Y TECNOLOGICA

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FOMENTO AL DESARROLLO CIENTIFICO Y TECNOLOGICO

1 RESUMEN EJECUTIVO

Describa en no más de una página la problemática u oportunidades que llevó a formular este proyecto (origen), su desarrollo, los resultados logrados y la proyección del mismo.

El objetivo principal del proyecto es desarrollar herramientas, aplicaciones y servicios computacionales, que aprovechen en forma efectiva las tecnologías asociadas a Internet para modernizar el sistema financiero nacional apoyando una mejor gestión de carteras de inversión en activos transados en el mercado nacional e internacional. Los desarrollos se focalizan preferentemente en la problemática de los fondos de pensiones, pero sus resultados impactan la gestión de otras carteras de inversión como las administradas por compañías de seguros y fondos mutuos. Esta modernización se apoya tanto en el estado del arte metodológico mundial como en investigación científica que aborde la problemática de mercados financieros poco profundos como el nacional, con activos que se transan con una baja frecuencia (*thin markets*), lo que dificulta el uso de numerosos procedimientos y metodologías utilizadas en los mercados desarrollados.

De este modo se pretende (1) apoyar una gestión más eficiente de las carteras al incluirse mayor información relativa a retornos y riesgos involucrados, (2) hacer un análisis de estrategias de inversión que apoye la asignación de activos (*asset allocation*), (3) apoyar funciones de medición y gestión del riesgo y (4) establecer un conjunto de benchmarks para diversas carteras de inversión. Todo lo anterior busca favorecer la gestión e información para directivos y usuarios y, en ultimo término, la competitividad y desempeño de la industria.

El impacto económico y social de este proyecto es extremadamente alto, considerando que sólo el sistema de AFPs administra más de 100 billones de dólares, por lo que incrementos marginales en las rentabilidades, que se derivarían de la disponibilidad de mejores herramientas e información para gestionar los fondos y controlar sus riesgos, crearían una gran riqueza que sería capturada en su gran mayoría por los afiliados, aumentando el bienestar de los pensionados. Asimismo, este incremento en el valor de los fondos impulsaría el desarrollo de la economía nacional, aumentando el ahorro nacional y mejorando la asignación de recursos. La evaluación social reconoce por una parte que hay una demanda insatisfecha por tecnologías y servicios de gestión del riesgo y por otra la inexistencia de productos en el mercado internacional que aborden la problemática específica del mercado nacional.

El proyecto generó múltiples resultados en ámbitos económico-sociales, científico-tecnológicos, e institucionales de gran impacto.

En primer lugar dio origen a 3 nuevos servicios de gestión del riesgo que se distribuyen por Internet a través de RiskAmerica: Servicio SVC, (que valoriza diariamente instrumentos de renta fija con pocas transacciones), Servicio Índices (que entrega benchmarks con el comportamiento del mercado para diversas clases de activos) y Servicio Portfolio (que entrega información de retorno, riesgo y performance para carteras del mercado y propias). Estos servicios ya están siendo utilizados por muchas instituciones financieras del país.
Además el proyecto generó publicaciones en revistas y presentaciones en congresos internacionales, y apoyó la realización de tesis de magíster y memorias de título, fortaleciendo las actividades del FINlabUC-Laboratorio de Investigación Avanzada en Finanzas de la Pontificia Universidad Católica de Chile.

ABSTRACT.

The main objective of this project is to develop tools, software applications and services that use Internet technologies to provide better portfolio management for domestic and world market assets and modernize the Chilean financial markets. Although the new developments are primarily focused on pension funds, results can also be used for other portfolios like mutual funds or those managed by insurance companies. This market modernization is supported by standard state-of-the-art methodologies as well as new specific research on thin markets, a characteristic of Chilean financial markets which imposes difficulties in the use of many methodologies common in developed markets.

The goals of the project are (1) to induce a more efficient portfolio management which uses better risk and return information, (2) to analyze investment strategies and support the asset allocation process, (3) to improve risk management and measurement, (4) to define a set of portfolio benchmarks; thus, the goal is to help portfolio owners and managers, by providing better information and management tools, and improving industry competition and performance.

The economic and social impacts of this project are extremely high, considering that the AFP-system manages over US$ 100 billions. Thus, marginal increases on returns, due to better tools and information for managing funds and controlling risks, create a huge wealth, most of it transferred to fund members, increasing their welfare. This return increase would in turn stimulate the economy, increasing savings and improving resource allocation. The social project evaluation recognizes that there is an unsatisfied demand for technologies for risk management and that there are no available products in international markets that satisfy national market requirements.

The project had multiple results with great impact in economic-social, scientific-technological and institutional scope.

First it originated three new risk management services distributed by RiskAmerica using Internet: SVC Service (which prices fixed-income instruments with few market transactions), Indices Service (which provides benchmarks on market behavior of several asset classes), and Portfolio Service (which provides risk, return and performance information for market and private portfolios). These services are already been used by many financial institutions in Chile.

The project also generated journal publications and congress presentations and provided support for master´s and undergraduate thesis, strengthening the activities of the FINlabUC-Laboratorio de Investigación Avanzada en Finanzas de la Pontificia Universidad Católica de Chile.

2 SINTESIS DE RESULTADOS DEL PROYECTO

RESULTADO	cantidad	RESULTADO	cantidad
Importancia económica-social			
Nuevos productos		Productos mejorados	
Nuevos procesos		Procesos mejorados	
Nuevos servicios	**3**	Servicios mejorados	
Contratos empresas productoras		Convenios (contratos) para proyecto de escalamiento.	
Patentes		Marcas	
Registro de variedades vegetales		Otros registros de propiedad	
Otro (especificar)		*Otro (especificar)*	
Importancia científica-tecnológica			
Capacidades científico-tecnológicas obtenidas: -*Capacidad de desarrollar metodologías e implementar soluciones en el ámbito de la Ingeniería Financiera y Gestión del Riesgo*			
Otros Resultados C&T	**cantidad**	**Otros Resultados C&T**	**cantidad**
Artículos revista nacional, ISSN		Artículos revista nacional	
Artículos revista internacional, ISSN	**3**	Artículos revista internacional	
Artículos revista nacional, ISI		Capítulos libro nacional, ISBN	
Artículos revista internacional, ISI	**3**	Capítulos libro nacional	
Libros publicación nacional		Capítulos libro internacional, ISBN	
Seminarios nacionales*1		Capítulos libro internacional	
Seminarios internacionales*2	**1**	Libros publicación internacional	
Congresos nacionales*1		Proyectos I&D	
Congresos internacionales*2		Tesis doctorales	
Simposios nacionales*1		Tesis magister	**4**
Simposios internacionales*2		Postdoctorados	
Cursos*		Proyectos de títulos	**3**
Reconocimientos de laboratorio		Talleres*	
Presentaciones en Congresos Internacionales	**9**	*Otro (especificar)*	
Importancia ambiental			
Propuestas de normativa			
Otros (especificar)		*Otro (especificar)*	

**1.realizados por el proyecto, con expositores y/o ponencias nacionales.*
**2:realizados por el proyecto, con expositores y/o ponencias de extranjeros.*

3 CAPACIDADES CIENTIFICO-TECNOLOGICAS, PRODUCTOS Y SERVICIOS DESARROLLADOS POR EL PROYECTO. .

Durante la realización del proyecto se desarrolló la capacidad de desarrollar metodologías e implementar soluciones en el ámbito de la Ingeniería Financiera y Gestión del Riesgo.

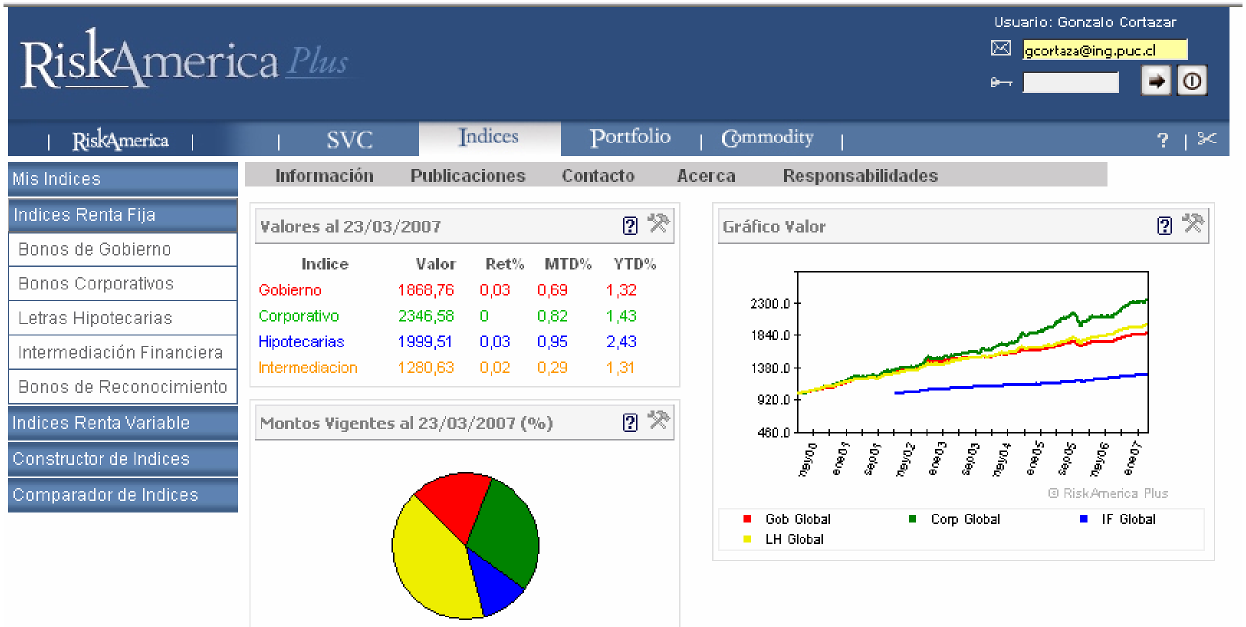
Los tres principales servicios ya desarrollados se distribuyen vía Internet como módulos independientes de la plataforma RiskAmericaPlus. Estos son:

Servicio 1: Módulo SVC

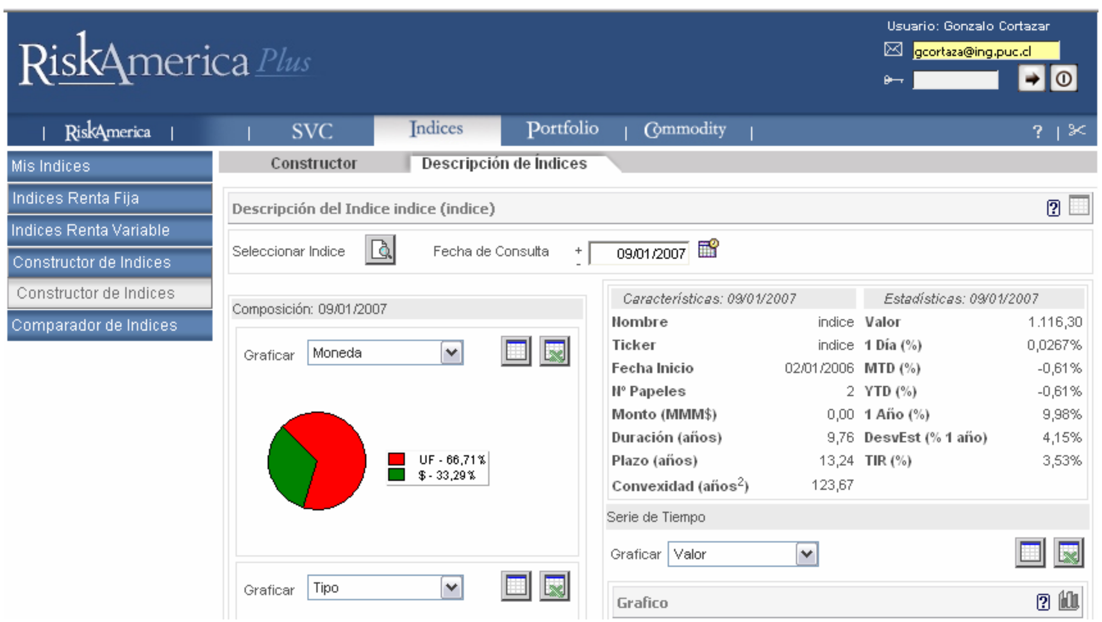
El SVC o Sistema de Valorización de Carteras consiste en un módulo del servicio RiskAmericaPlus cuyo objetivo es asignarle una TIR a cada nemotécnico solicitado.
El usuario envía vía Web un archivo indicando los nemotécnicos asociados a su cartera, devolviendo el sistema la TIR que el modelo le asigna a cada uno. La TIR del modelo depende de si el activo fue transado ese día, de cuál es la curva de referencia para el día (la que es actualizada diariamente) y de la historia de spreads que este nemotécnico ha exhibido respecto de la curva en el pasado.

Servicio 2: Módulo Índices

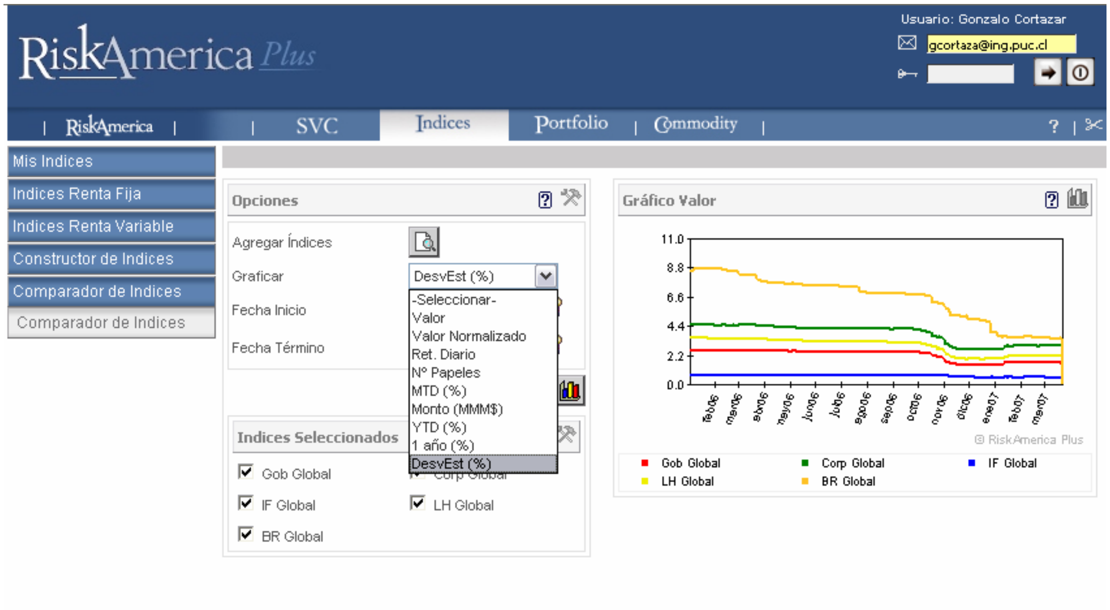
Este módulo entrega información referida al comportamiento del mercado financiero. Esta descripción se realiza en términos de distintas familias y clases de activos, incluyéndose tanto renta fija como variable.



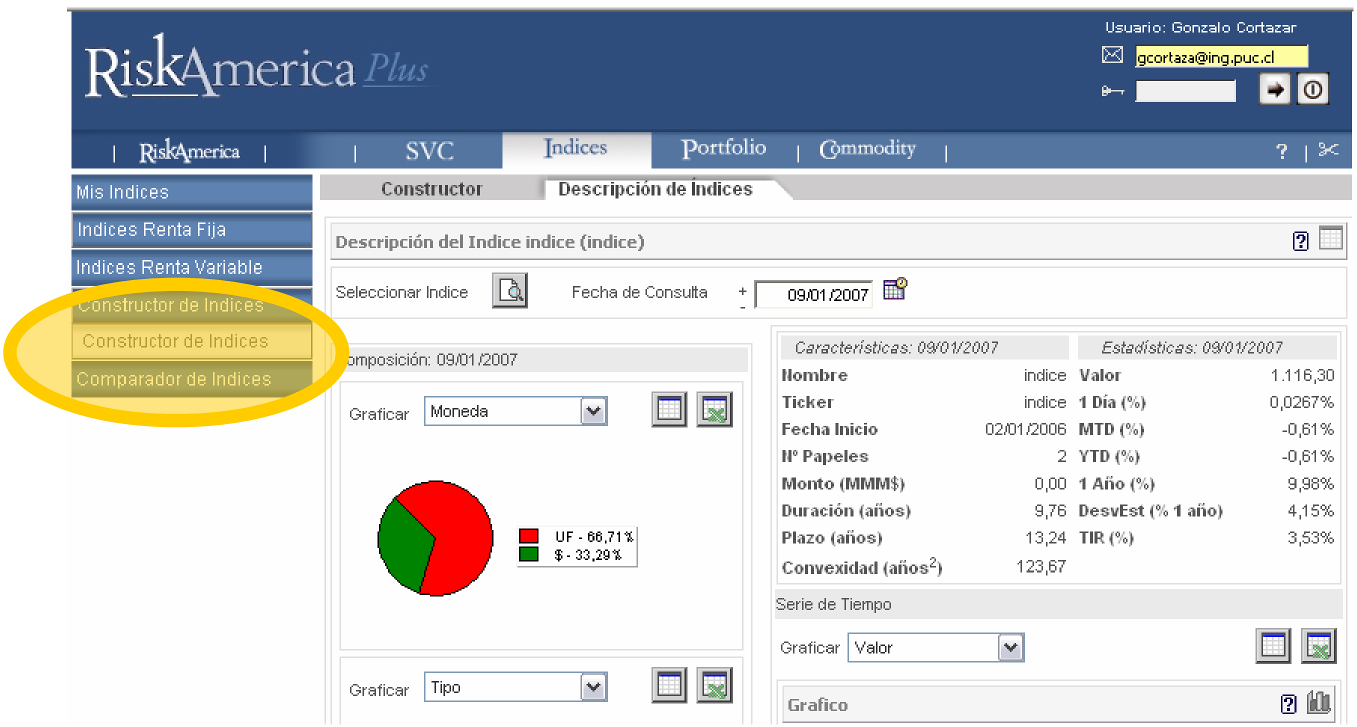
Para cada uno de los más de 100 índices existentes se entrega información de su composición así como de su comportamiento en términos de retornos y riesgos.



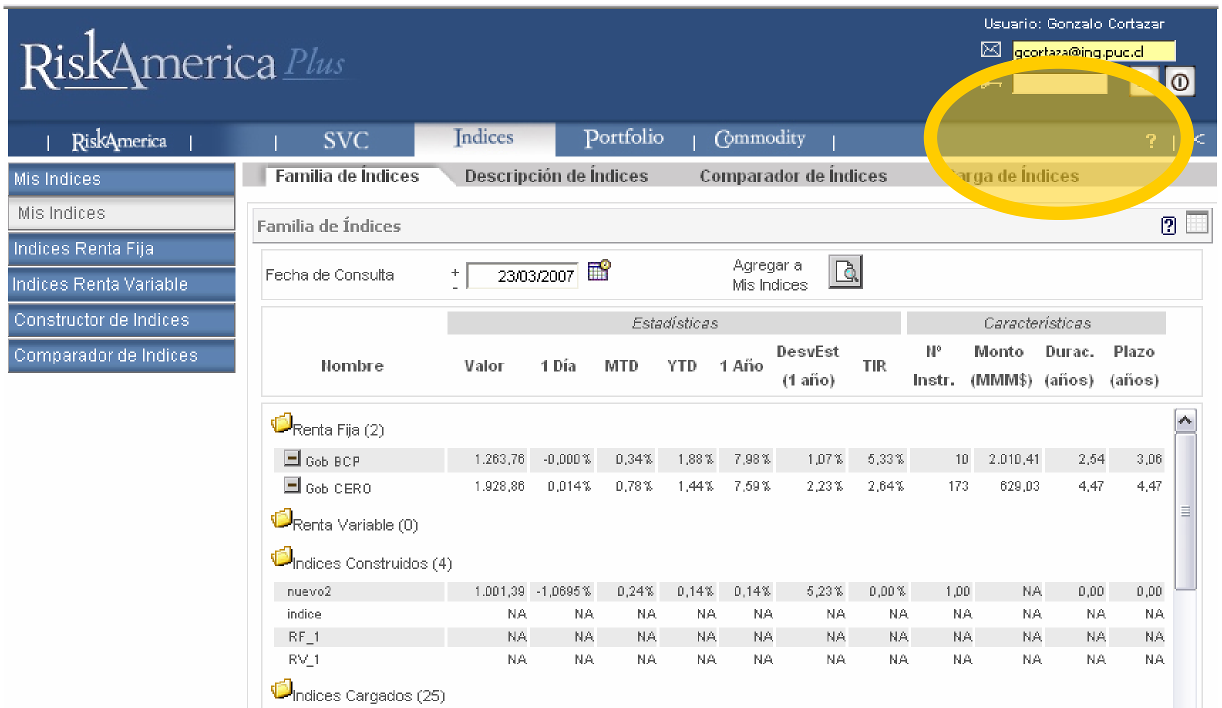
Asimismo, se pueden comparar y realizar análisis entre los distintos índices:



Se entrega además la posibilidad de construir y índices personalizados:



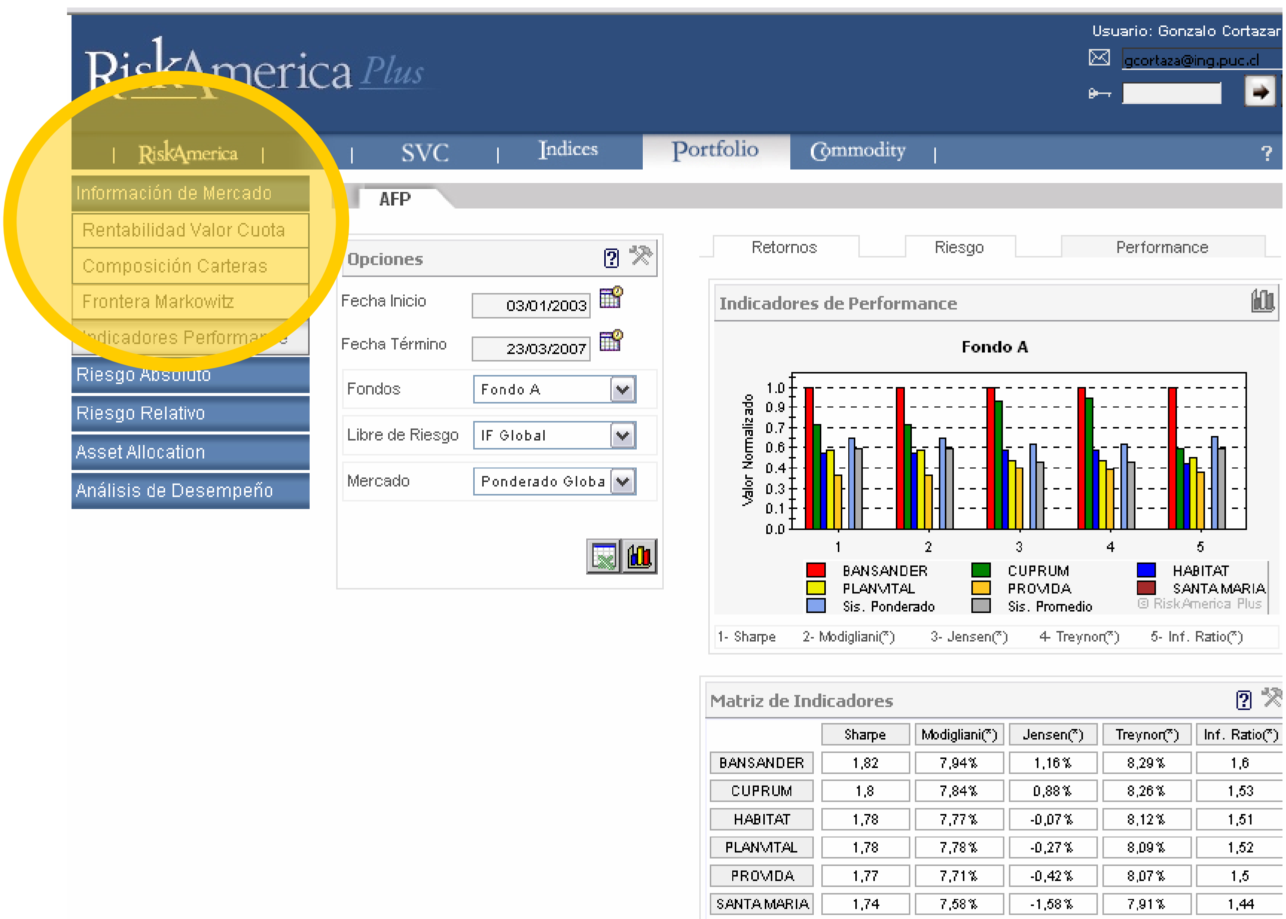
También se pueden cargar índices generados externamente



Servicio 3: Módulo Portfolio

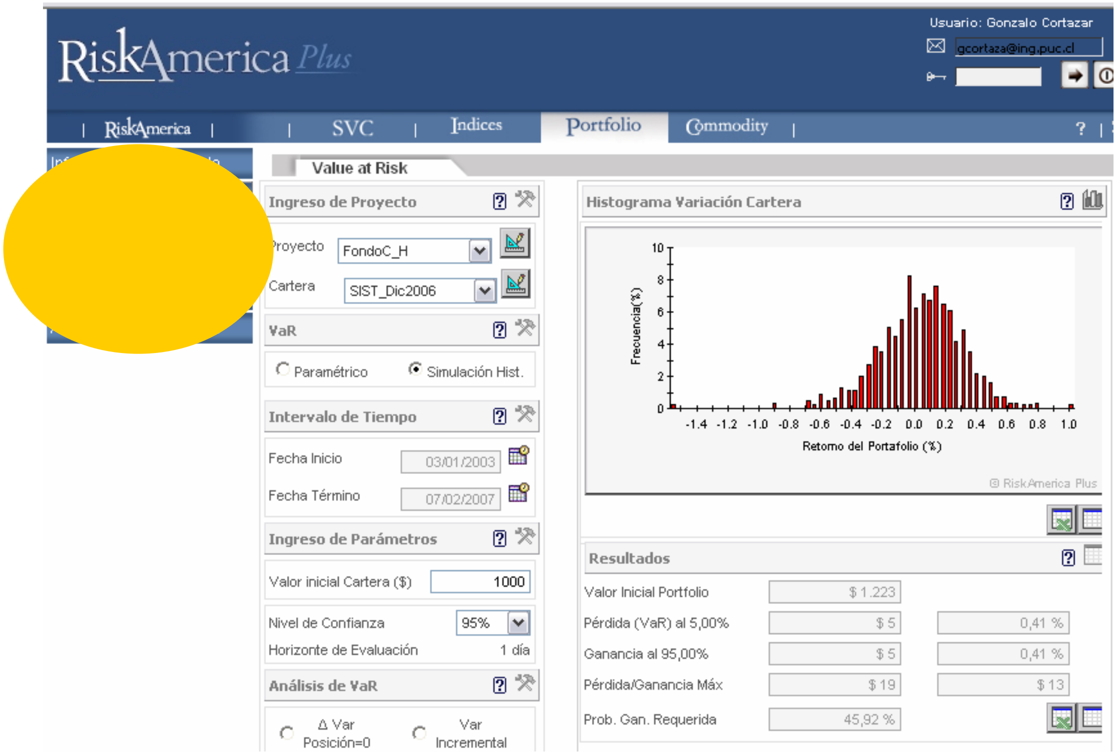
Este módulo entrega información de de riesgo, retorno y performance tanto de carteras existentes del mercado, como de carteras propias.

En *Información de Mercado* entrega información información de riesgo, de retornos y de performance de carteras públicas de AFP y de Fondos Mutuos, como se muestra en las siguientes páginas web:

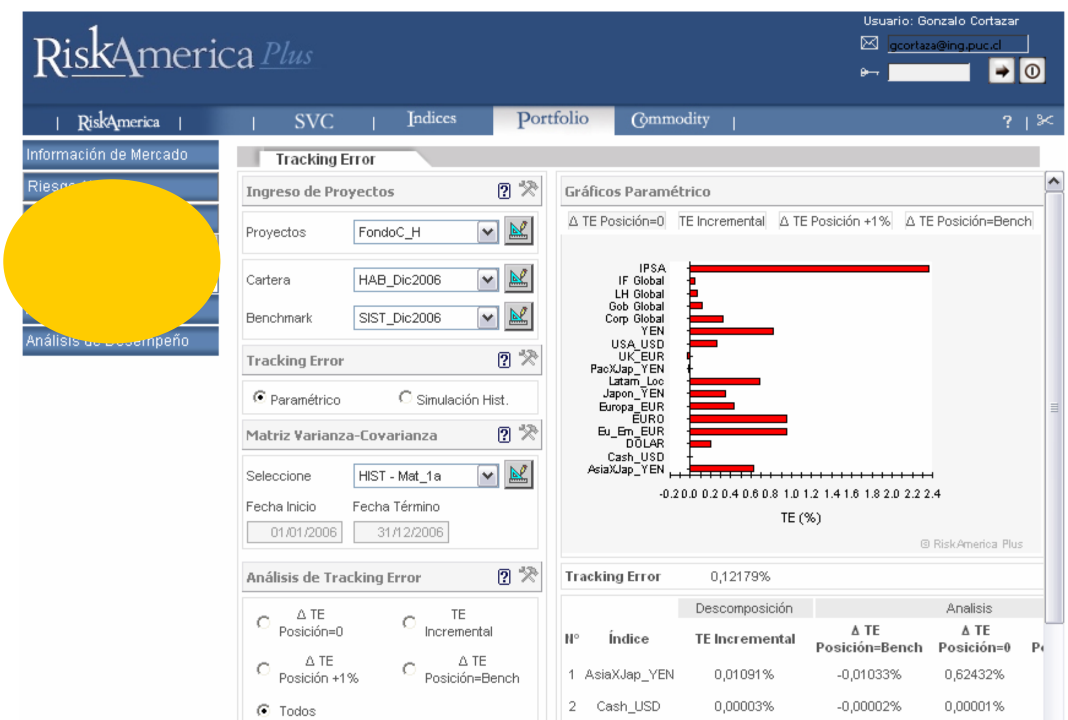


	Sharpe	Modigliani(*)	Jensen(*)	Treynor(*)	Inf. Ratio(*)
BANSANDER	1,82	7,94%	1,16%	8,29%	1,6
CUPRUM	1,8	7,84%	0,88%	8,26%	1,53
HABITAT	1,78	7,77%	-0,07%	8,12%	1,51
PLANVITAL	1,78	7,78%	-0,27%	8,09%	1,52
PROVIDA	1,77	7,71%	-0,42%	8,07%	1,5
SANTA MARIA	1,74	7,58%	-1,58%	7,91%	1,44

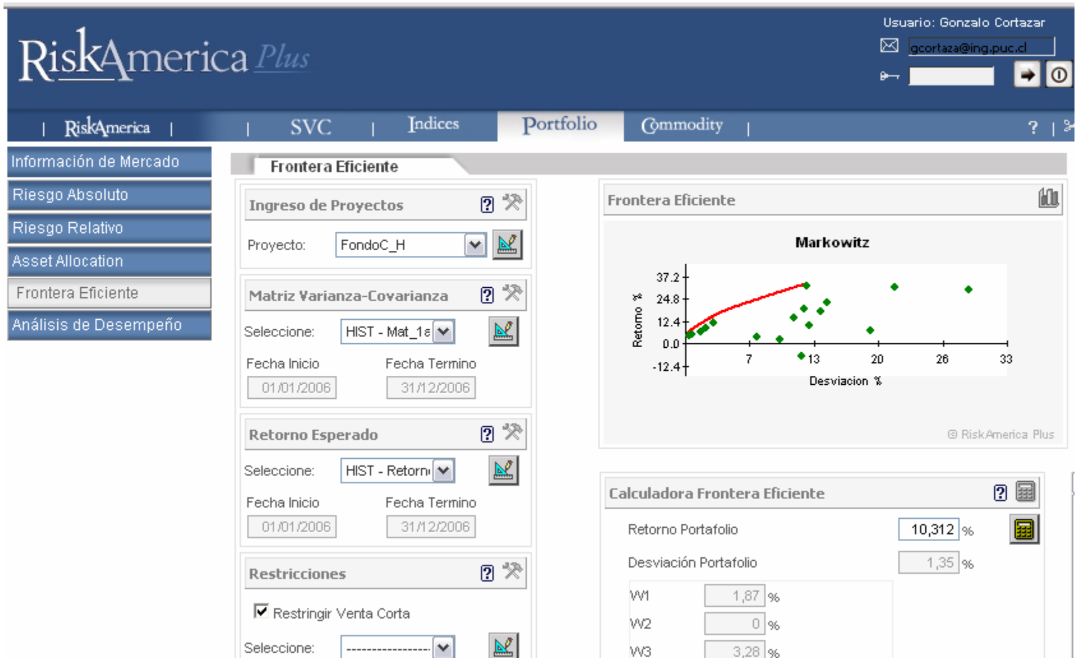
En *Riesgo Absoluto* se entregan herramientas para calcular el Value-at-Risk de carteras propias tanto por métodos paramétricos como por simulación histórica.



En *Riesgo Relativo* se entregan herramientas para calcular el Tracking Error y el VaR Relativo a entre dos carteras.



En *Asset Allocation* se entregan herramientas para calcular carteras con combinaciones riesgo-retorno óptimas.



En *Análisis de Desempeño* se entregan herramientas para calcular carteras el performance de carteras definidas por el usuario.

RiskAmerica Plus

RiskAmericaPlus busca proveer un conjunto de servicios enfocados a satisfacer las necesidades específicas del mercado financiero chileno y apoyar de una manera integral la gestión y decisiones de inversión de los distintos actores del mercado.

Hasta la fecha, la investigación financiera mundial se ha centrado principalmente en comprender y modelar el comportamiento de los mercados desarrollados, donde se concentran los mayores volúmenes de inversión. Sin embargo los mercados emergentes, debido a su menor tamaño, han sido relegados a un segundo plano en esta materia, generándose así una escasez de herramientas que se ajusten correctamente a la realidad específica de estos mercados.

Considerando esta necesidad, la Pontificia Universidad Católica de Chile a través de su Laboratorio de Investigación Avanzada en Finanzas, FINlabUC, ha creado RiskAmericaPlus y desarrollado Ingeniería Financiera orientada específicamente a resolver problemas relevantes para el mercado nacional.

El desarrollo de RiskAmericaPlus ha sido llevado a cabo por la Universidad Católica de Chile y apoyado por CONICYT a través de FONDEF, Banco Santander, AFP Habitat y Fundación COPEC-Universidad Católica, logrando de esta manera la combinación necesaria entre excelencia académica y experiencia de mercado, que han permitido desarrollar una completa familia de herramientas acordes a las necesidades reales y con la robustez teórica de los mercados más exigentes.

CONICYT COMISIÓN NACIONAL DE INVESTIGACIÓN CIENTÍFICA Y TECNOLÓGICA

Santander GLOBAL BANKING & MARKETS

AFP HABITAT Seguridad y Confianza

FUNDACIÓN COPEC • UNIVERSIDAD CATÓLICA

Servicios RiskAmerica Plus

RiskAmerica
Estimaciones de estructuras de tasas de interés y herramientas de valorización y análisis para los distintos instrumentos y transacciones del mercado de renta fija nacional.

SVC
Servicio diario de valorización y asignación de tasas para los distintos instrumentos de renta fija del mercado nacional.

Índices
Familia de índices del mercado nacional que permite tener referentes adecuados para la medición y comparación de rendimientos de las distintas carteras de inversión.

Portfolio
Completa gama de herramientas que permiten hacer más eficiente la gestión integral de carteras de inversión en Chile: Asignación de Activos, Gestión del Riesgo y medición de Performance.

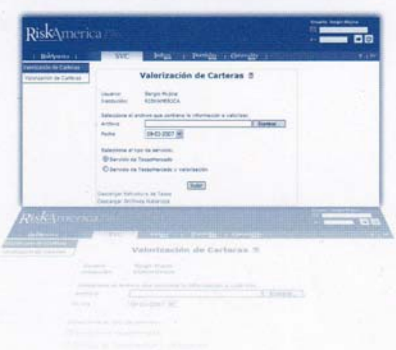
Commodity
Ingeniería Financiera para la evaluación y gestión eficiente de proyectos y activos expuestos a las variaciones de los precios de commodities.

www.riskamerica.com / Fono: (56 2) 354 4086 - 354 4823 / Fax: (56 2) 552 1608 / E-mail: info@riskamerica.com

SVC

Nuestro Servicio de Valorización de Carteras, SVC, consiste en un servicio diario de valorización y asignación de tasas para los distintos instrumentos de renta fija del mercado nacional.

El mercado chileno de instrumentos de renta fija presenta una baja frecuencia de transacciones, lo que dificulta la obtención diaria de precios de mercado para los distintos activos que componen las carteras de renta fija nacional. Diariamente sólo el 0,5% de los instrumentos del mercado presentan transacciones, por lo que se hace necesario contar con una metodología que permita estimar las tasas de los nemotécnicos restantes.



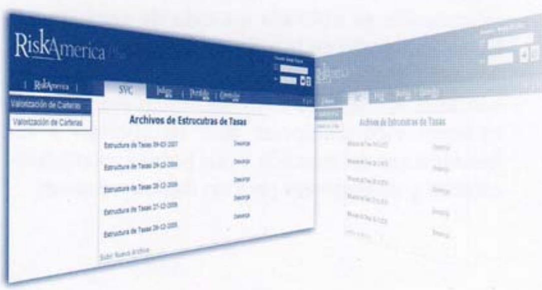
Con el fin de solucionar estos problemas y de proporcionar herramientas que apoyen las decisiones de inversión y gestión del riesgo financiero, hemos desarrollado un modelo de estimación de tasas para los instrumentos de renta fija del mercado nacional.

Este modelo se caracteriza por maximizar el uso de la información de mercado disponible, a través de un registro de transacciones diario e histórico. Esta información es utilizada a través de consultas de spreads así como en la estimación de Estructuras de Referencia mediante modelos dinámicos de no arbitraje aplicados al mercado nacional, los cuales permiten obtener valorizaciones que se ajustan a los movimientos del mercado y que poseen volatilidades estables y consistentes para los distintos papeles.

Servicio de Valorización de Carteras

El Servicio de Valorización de Carteras consiste en una aplicación web a través de la cual nuestros usuarios pueden cargar diariamente sus carteras y descargarlas valorizadas en un plazo no mayor a 5 minutos.

Las tasas de valorización son actualizadas diariamente a partir de la información de transacciones de la Bolsa de Comercio de Santiago, quedando accesibles para nuestros usuarios a partir de las 14:45 horas. Es posible valorizar carteras en horario continuado y también para fechas pasadas.



El servicio consiste en una valorización integral del instrumento entregando toda la información generada referente a éste:

- Tasa de Valorización
- Precio en Porcentaje del Valor Par
- Precio en pesos
- Plazo al Vencimiento
- Duración
- Convexidad
- Clasificación de Riesgo
- TIR Base
- Spread
- Clase de Valorización

Se incluye el Precio en pesos del instrumento, que puede ser evaluado para una posición estándar, permitiéndole así a nuestros usuarios ponderarlo por las posiciones reales de su cartera.

Adicionalmente se entrega la Clase de Valorización, lo que otorga un mejor control sobre la información y la capacidad de poder monitorear eficientemente las tasas si así se desea.

Indices

Nuestro servicio Indices entrega una completa familia de índices del mercado nacional, proveyendo así referentes adecuados para la medición y comparación de rendimientos de las distintas carteras de inversión.

Los mercados desarrollados poseen cientos de indicadores que permiten monitorear sus comportamientos, realizar análisis históricos y comparar los rendimientos relativos de las distintas carteras de inversión, mejorando así la capacidad de análisis y fomentando el crecimiento y profundidad de estos.

Con el objetivo de fomentar el crecimiento y transparencia del mercado chileno y proveer indicadores confiables y precisos del comportamiento de éste, hemos construido una completa familia de índices, la cual representa el mercado nacional en su totalidad.

Estos índices son construidos en base a los precios de nuestro Servicio de Valorización de Carteras, los cuales siguen diariamente los movimientos de mercado y poseen volatilidades estables y consistentes para los distintos papeles, lo que garantiza la estabilidad y correcto comportamiento de los distintos índices.

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Herramientas
Indices

Índices
Completa familia de índices, que provee una cobertura total del mercado nacional a través de índices agregados y detallados para los distintos instrumentos, plazos, clasificaciones de riesgo y monedas del mercado local:

- Bonos de Gobierno
- Bonos Corporativos
- Letras Hipotecaras
- Intermediación Financiera
- Acciones

Comparador de Índices
Herramienta que permite comparar y descargar características y estadísticas de los distintos índices a lo largo del tiempo.

Constructor de Índices
Herramienta que permite crear índices propios, considerando conjuntamente instrumentos específicos e índices de mercado. Permitiendo de esta manera a nuestros usuarios construir y monitorear diariamente índices que reflejen sus carteras de inversión o cualquier cartera que deseen representar.

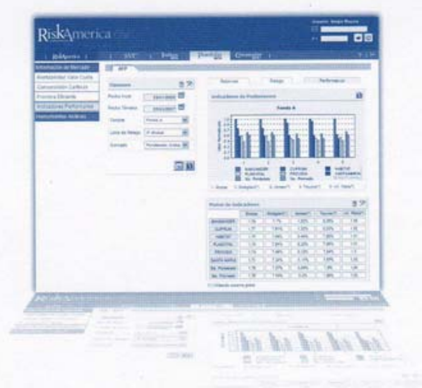
Carga de Índices
Adicional a la construcción de índices propios, los usuarios pueden cargar índices para así poder compararlos y utilizarlos como inputs de las distintas herramientas de RiskAmericaPlus.

Mis Índices
Herramienta que le permite a los usuarios manejar una selección propia de índices, construidos, cargados o provistos por RiskAmerica, y trabajar diariamente con estos a través de distintas herramientas de análisis.

Portfolio

Nuestro servicio Portfolio provee una completa gama de herramientas que permiten hacer más eficiente la gestión integral de carteras de inversión en Chile: Asignación de Activos, Gestión del Riesgo y Medición de Performance.

En el competitivo mercado financiero actual, el manejo adecuado de una cartera de inversión requiere un monitoreo constante de la composición, comportamiento y exposición de ésta, así como la comparación de su desempeño con el de otras carteras de mercado.



Con el fin de facilitar estás tareas y apoyar integralmente las decisiones de inversión y gestión de carteras, hemos desarrollado una completa familia de herramientas relacionadas con las áreas de Asset Allocation, Análisis de Riesgo y Medición de Performance. Estás herramientas le permiten a nuestros usuarios cuantificar y monitorear de una manera rápida y fácil el desempeño y exposición de sus carteras, y a la vez diseñar y evaluar eficientemente las distintas estrategias de inversión.

Adicionalmente, estas herramientas pueden ser usadas en conjunto con nuestra familia de índices, permitiendo a nuestros usuarios representar adecuadamente el mercado financiero local y gestionar así de manera eficiente y confiable sus carteras de inversión.

Herramientas
Portfolio

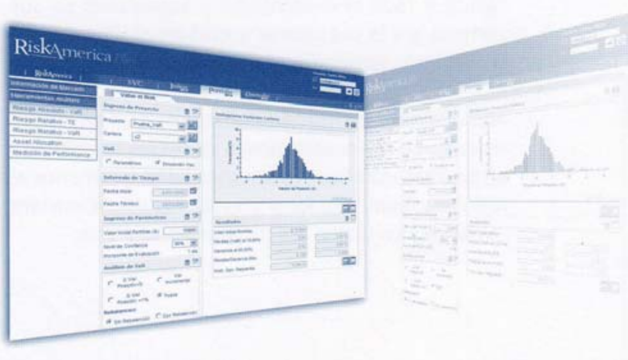
Información de Mercado
Conjunto de herramientas que permite estudiar en detalle distintas carteras públicas del mercado financiero nacional, entregando información de rentabilidades, composición y performance.

Medición de Riesgo Absoluto
Herramienta que permite calcular y realizar distintos análisis de Value at Risk a las carteras construidas por el usuario.

Medición de Riesgo Relativo
Herramientas que permiten calcular y realizar distintos análisis de Tracking Error y Value at Risk Relativo a carteras y benchmarks construidos por el usuario.

Asset Allocation
Herramienta que permite realizar análisis de Asset Allocation para conjuntos de activos definidos por el usuario.

Medición de Performance
Conjunto de herramientas que permiten analizar y comparar el desempeño de carteras, entregando indicadores de retorno, riesgo y performance para un periodo de tiempo específico.



4 RESULTADO DE EVALUACIÓN EX-POST DESARROLLADA POR LA INSTITUCIÓN

Tal como se plantea en la Formulación del proyecto, el beneficio social principal de este proyecto se genera al contribuir a la modernización del mercado financiero nacional. Un resultado exitoso en cuanto a nuevas herramientas de gestión de carteras, como es el caso de lo ocurrido con este proyecto debiera inducir que las administradoras de fondos de pensiones, pueden mejorar la gestión de sus carteras y de esta manera obtener mayores retornos de sus inversiones, sin incrementar el riesgo asumido.

El principal impacto económico social del proyecto es el **Incremento Marginal de la Rentabilidad** aplicado a una **Fracción** de los **Fondos Potenciales** que pudieran verse beneficiados con las herramientas de gestión de carteras desarrolladas. Por último, se debe estimar el **adelantamiento** de los flujo**s** (en número de años) que representa la realización del proyecto comparado con la situación base sin proyecto (i.e. se estima que si no hubiera habido proyecto, otro similar se hubiera desarrollado teniendo el mismo impacto después).

Los cuatro parámetros anteriores son difíciles de estimar y de ellos depende el resultado de los indicadores económicos-sociales.

En la **Evaluación ex–ante** presentada en la formulación del proyecto se asumieron los siguientes valores para estos parámetros:

Incremento Marginal de la Rentabilidad = 0,25%

Fracción = 25% de las administradoras* 60% de los activos = 15%

Fondos Potenciales = MMUS$ 40.000

Adelantamiento = 1 año (si no se hace el proyecto los flujos se realizan 1 año después).

Con lo que el beneficio económico-social por año se estimó en US$ 15 millones.

Se puede señalar que desde la formulación del proyecto uno de los parámetros se ha incrementado significativamente, al aumentar el parámetro **Fondos Potenciales** subió al año 2007 a más de **MM US$100.000**, es decir **2,5** veces mayor que el valor estimado en la Formulación. Esto permitiría dividir por 2,5 alguno de los indicadores anteriores (por ejemplo suponer que el **Incremento Marginal de la Rentabilidad** fuera 0,1 en vez de los 0,25 asumidos originalmente) y conservar el valor de los indicadores originales.

Debido a lo anterior, una evaluación conservadora mantendría los indicadores económico-sociales presentados en la Formulación inicial, siendo sus flujos netos:

.

FLUJO NETO

Ingresos	0	0	10.754	238	238	119	119	0	0	0
Costos	0	0	276	136	136	70	71	38	38	0
Inversión	0	-509	-509	-6	-6	-6	-6	-6	-6	-6
Costo total I+D	290	219	0	0						
Fondef	97	97	0	0						
Beneficio neto	-290	290	10.987	108	108	55	54	-32	-32	6

TIR	**568,06%**
VAN (10%) MM$	**9.244**
VAN/VAI	**18,90**
VAN/FONDEF	**50,14**

Nota: M/N = moneda nacional
M/E = moneda extranjera

.

GOBIERNO DE CHILE
CONICYT

III PARTE. INFORME DE GESTIÓN

Código Proyecto: D03I1039

Nombre del Proyecto: DESARROLLO DE HERRAMIENTAS COMPUTACIONALES PARA OPTIMIZAR LA GESTION DE CARTERAS DE INVERSION EN MERCADOS EMERGENTES: APLICACION A LOS FONDOS DE PENSIONES EN CHILE

Instituciones Participantes: Pontificia Universidad Católica de Chile

Otros Participantes: AFP Habitat S.A.
RiskAmerica- Dictuc S.A.

Director del Proyecto: .Gonzalo Cortazar Sanz, Firma:.......

Fecha de emisión : 23/07/2007

FONDEF
FOMENTO AL DESARROLLO CIENTIFICO Y TECNOLOGICO

COMISION NACIONAL DE INVESTIGACION CIENTIFICA Y TECNOLOGICA
BERNARDA MORIN 495 • CASILLA 297-V• CORREO 21• FONO: 3654400 • FAX: 6551394 • CHILE

1. OBJETIVOS DEL PROYECTO

1.1 OBJETIVOS GENERALES

Programados

El objetivo principal del proyecto es desarrollar herramientas, aplicaciones y servicios computacionales, que aprovechen en forma efectiva las tecnologías asociadas a internet para modernizar el sistema financiero nacional apoyando una mejor gestión de carteras de inversión en activos transados en el mercado nacional e internacional.

Los desarrollos se focalizarán preferentemente en la problemática de los fondos de pensiones, pero sus resultados impactarán la gestión de otras carteras de inversión como las administradas por compañías de seguros y fondos mutuos, entre otros.

Esta modernización se apoyará tanto en el estado del arte metodológico mundial como en investigación científica que aborde la problemática de mercados financieros poco profundos como el nacional, con activos que se transan con una baja frecuencia (thin markets), lo que dificulta el uso de numerosos procedimientos y metodologías utilizadas en los mercados desarrollados.

De este modo se pretende (1) apoyar una gestión más eficiente de las carteras al incluirse mayor información relativa a retornos y riesgos involucrados, (2) hacer un análisis de estrategias de inversión que apoye la asignación de activos (asset allocation), (3) apoyar funciones de medición y gestión del riesgo y (4) establecer un conjunto de benchmarks para diversas carteras de inversión. Todo lo anterior busca favorecer la gestión e información para directivos y usuarios y, en ultimo término, la competitividad y desempeño de la industria.

No obtenidos

Obtenidos no programados

*Aún cuando desde un principio se esperaba no restringir el impacto del proyecto sólo a la industria de las AFP sino también a la de otros actores de la industria financiera, el impacto sobre la industria de los **Fondos Mutuos** fue más fuerte y más anticipado de lo programado.*
*Es así como **desde marzo de 2006** los precios para todos los instrumentos de renta fija son entregados vía Internet a todos los fondos mutuos nacionales como precio oficial para ser utilizados en el cálculo diario de la cuota de todos los fondos mutuos, según circular de la Superintendencia de Valores y Seguros y en acuerdo con la Asociación de Administradoras de Fondos Mutuos de Chile. Asimismo, ya se han iniciado contactos internacionales que pueden llevar a exportar algunas de las tecnologías desarrolladas.*

1.2 OBJETIVOS ESPECIFICOS

Programados

El proyecto en sus diversos ámbitos de acción tiene los siguientes objetivos específicos:

1) **Generar conocimiento científico** acerca del comportamiento de los mercados financieros nacionales y su interrelación con los mercados internacionales. Esta investigación deberá generar información de interés para Chile y para entender el comportamiento de los mercados en los diversos mercados emergentes.

2) **Realizar desarrollos tecnológicos** que se expresen en nuevas metodologías y en herramientas, aplicaciones y servicios computacionales que tengan un impacto significativo en el manejo de los recursos de los fondos de pensiones nacionales. Este impacto se producirá a través de: a) la optimización de las carteras de inversión de las administradoras sujetas a las regulaciones existentes, y b) de mejoramientos sistémicos (a nivel de la industria) producto del mayor conocimiento de los riesgos asociados a los diversos activos, lo que debiera permitir ajustar regulaciones y restricciones de inversión de modo de poder limitar las exposiciones al riesgo deseadas incurriendo en un menor costo en términos de rentabilidad.

3) **Generar información** en la forma de indicadores de gestión comparativa que sea objetiva y confiable y que apoye la modernización, transparencia y desarrollo del mercado financiero nacional.

4) **Desarrollar mecanismos de transferencia** efectiva de resultados de modo de maximizar el impacto sobre el sistema productivo del país y las oportunidades comerciales del proyecto asegurando su sustentabilidad.

5) **Formar investigadores y profesionales especializados** en aplicaciones financieras de alto nivel. Una condición necesaria para sustentar el desarrollo de la industria de las aplicaciones financieras en Chile, y de colaborar de ese modo a modernizar el mercado financiero, es contar con el capital humano capacitado que actúe tanto como oferente y como contraparte de estas aplicaciones.

No obtenidos

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Obtenidos no programados

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2. RESULTADOS DEL PROYECTO

2.1 PRODUCTOS, SERVICIOS Y/O PROCESOS. *Detalle según tabla.*

NOMBRE		DESCRIPCIÓN	TIPO DE RSULTADO P: PROGRAMDO I: INESPERADO	TIPO DE INNOVACIÓN *	ESTADO DEL DESARROLLO**	MEDIDAS DE PROTECCIÓN***
1	Módulo **SVC**	Módulo Computacional Vía WEB que Valoriza Instrumentos de Renta Fija (PortfolioValue)	P	Nuevo Servicio	***Comercializción***	***No es posible de ser patentado***
2	Módulo **Indices**	Módulo Computacional Vía WEB que Entrega Información de Referencia de Mercado (PortfolioBenchmarks)	P	Nuevo Servicio	***Comercializción***	***No es posible de ser patentado***
3	Módulo **Portfolio**	Módulo Computacional Vía WEB que entrega herramientas de gestión para Carteras de Inversión (PortfolioRisk+AssetAllocation+RiskMatrix)	P	Nuevo Servicio	***Comercializción***	***No es posible de ser patentado***

2.2 PAQUETE TECNOLÓGICO

PRODUCTO, PROCESO o SERVICIO *(nombre)*	**PAQUETE TECNOLÓGICO** *(enumere el conjunto de elementos que compone el paquete tecnológico asociado al producto o proceso desarrollado)*
Módulo **SVC**	Set integrado de herramientas computacionales vía Internet orientadas a la valorización de instrumentos financieros
Módulo **Indices**	Set integrado de herramientas computacionales vía Internet orientadas a generar y entregar información relativa al comportamiento de los mercados financieros
Módulo **Portfolio**	Set integrado de herramientas computacionales vía Internet orientadas a apoyar la gestión de carteras de inversión.

2.3 COMPETITIVIDAD DE LOS PRODUCTOS, SERVICIOS Y/O PROCESOS MEJORADOS*(agregue más columnas si es necesario)*

PRODUCTO SERVICIO O PROCESO *(nombre)*	**VENTAJA***(naturaleza del beneficio)*	**MONTO** *(valor actual del beneficio en MM$)*	**PRODUCTO, SERVICIO O PROCESO CON QUE COMPITE** *(nombre, breve descripción)*	**DESVENTAJAS** *(exprese cuantitativamente)*
Módulo **SVC**	Calidad de Información	Valor mejor información usuario	Sistemas Internos/otros proveedores	Dependencia externa
Módulo **Indices**	Calidad de Información	Valor mejor información usuario	Sistemas Internos/ otros proveedores	Dependencia externa
Módulo **Portfolio**	Calidad de Herramientas	Valor mejor información usuario	Sistemas Internos/	Dependencia externa

2.4 MERCADO

PRODUCTO SERVICIO O PROCESO *(nombre)*	OFERTA ACTUAL Volumen *Indique unidades*	MM$ (2007)	DEMANDA ESTIMADA Volumen *Indique unidades*	MM$ (2007)	%DE PARTICIPACIÓN ESPERADA DE MERCADO	AÑOS *	CANALES DE COMERCIALIZACIÓN
Módulo **SVC**	240 Usuario-Mes	180	720 Usuario-Mes	240	60%	4	RiskAmerica
Módulo **Indices**	36 Usuario-Mes	10	720 Usuario-Mes	120	60%	4	RiskAmerica
Módulo **Portfolio**	18 Usuario-Mes	8	720 Usuario-Mes	210	60%	4	RiskAmerica

2.5 VALORIZACIÓN ECONÓMICA DE LOS RESULTADOS DEL PROYECTO *Indique en forma sintética. Los detalles y justificación incorpórelos en el plan de negocios)*

PRODUCTO, SERVICIO O PROCESO	VALOR ACTUAL (MM%)	CRITERIOS PARA LA FIJACIÓN DE PRECIO	MODALIDAD DE LA TRANSFERENCIA
SVC + Índices+ Portfolio	MM$781	No existen consideraciones especiales	Excedentes de Dictuc SA que es de propiedad de la PUC

2.6 TRANSFERENCIA TECNOLÓGICA

PRODUCTO SERVICIO O PROCESO	INSTITUCIÓN /EMPRESA PRODUCTORA *)*	PARTICIPÓ EN EL DESARROLLO DEL PROYECTO	CONTRATO DE TRANSFERENCIA, VENTA O LICENCIA.
SVC + Índices+ Portfolio	RiskAmerica	Si	MM$ 300

Continuación

PRODUCTO SERVICIO O PROCESO *(nombre)*	**ACCIONES FUTURAS** *(diga cuáles son las acciones, recursos y plazos, no desarrolladas, ni proveídos por el proyecto que permitirán la explotación de los resultados del proyecto)*	**COSTO** *(estimado en MM$ de 1999 de las acciones necesarias faltantes para la transferencia)*	**FUENTES DE FINANCIAMIENTO**
SVC + Índices + Portfolio	Contactos individuales y eventos de promoción		

2.7 VENTAS INSTITUCIONALES *(de productos y servicios del proyecto, no consideradas en el pto. 2.6)*

CONCEPTO *(qué se vendió)*	**AÑO** *(que se efectúo la venta)*	**MONTO** *(en MM$ del año 1999)*

2.8 CAPACIDADES TECNOLOGICAS. *Detalle las capacidades tecnológicas creadas o mejoradas con este proyecto. (Ejemplos en APENDICE·1)*

CAPACIDADES TECNOLÓGICAS		**PRODUCTOS O SERVICIOS QUE SE PUEDEN OBTENER DE LAS CAPACIDADES**	**USUARIOS DE LOS PRODUCTOS O SERVICIOS** .
Nº	***Nombre***	***Nombre del producto o servicio***	***Nombre del usuario***
1	*Capacidad de desarrollar metodologías e implementar soluciones en el ámbito de la Ingeniería Financiera y Gestión del Riesgo*	Nuevos Servicios y Capacitación en gestión del riesgo	Instituciones financieras y regulatorias

2.9 CAPACIDAD INSTITUCIONAL PARA GESTIÓN CIENTÍFICO- TECNOLÓGICA *(describa las principales capacidades creadas o fortalecidas por el proyecto, señalando expresamente cual es el caso)*

Fortalecimiento del FinlabUC-Laboratorio de Investigación Avanzada en Finanzas lo que permite abordar nuevos proyectos de investigación aplicada con nuevos desarrollos tecnológicos asociados.

2.10 FORMACIÓN DE PERSONAL

FORMACION CIENTIFICO-TECNOLOGICA DE PARTICIPANTES EN EL PROYECTO.

Nº	**Tipo** 1= Proyectos de títulos 2= Tesis de magister 3= Tesis doctorales 4= Posdoctorados	**Título de Curso o taller o del proyecto de título o de la tesis**	**Nombre del participante**	**RUT**	**Disciplina** (disciplina Fondecyt predominante)	**Institución o empresa** (en que realizó la actividad de formación; indíquelos todos)	**Mes y Año** (en que el participante logró la formación)	**Ciudad/País** (donde se realizó la formación; indíquelos todos)
1	2	"Modelo estocástico multicommodity para la dinámica de precios de contratos futuros. Selección y estimación del modelo utilizando componentes principales comunes y filtro de Kalman"	FELIPE SEVERINO DIAZ	14146082-K	Finanzas	Pontificia Universidad Católica de Chile	Marzo, 2007	Santiago-Chile
2	2	"Estimación de Spreads por Liquidez en un Mercado con Pocas Transacciones: El Caso del Mercado de Bonos del Banco Central de Chile"	PEDRO MATÍAS MORAL MESA	13551671-6	Finanzas	Pontificia Universidad Católica de Chile	Enero, 2006	Santiago-Chile
3	2	"Metodología e Implementación de Métodos de VALUE AT RISK en Mercados de Renta Fija con baja Frecuencia de	ALEJANDRO ADRIAN BERNALES SILVA		Finanzas	Pontificia Universidad Católica de Chile	Diciembre, 2005	Santiago-Chile

Nº	Tipo 1= Proyectos de títulos 2= Tesis de magister 3= Tesis doctorales 4= Posdoctorados	Título de Curso o taller o del proyecto de título o de la tesis	Nombre del participante	RUT	Disciplina (disciplina Fondecyt predominante)	Institución o empresa (en que realizó la actividad de formación; indíquelos todos)	Mes y Año (en que el participante logró la formación)	Ciudad/País (donde se realizó la formación; indíquelos todos)
		Transacciones”						
4	2	“Modelos Estocásticos de Precios de Commodities y Estimación Conjunta de la Dinámica de dos Commodities Mediante el Filtro de Kalman”	CARLOS IGNACIO MILLA GONZALEZ	13922911-8	Finanzas	Pontificia Universidad Católica de Chile	Diciembre, 2005	Santiago-Chile
5	1	“Bonos Corporativos: una Revisión del Mercado y una Aproximación a un Método Práctico de Valorización”	CLAUDIO EDUARDO HELFMANN SOTO		Finanzas	Pontificia Universidad Católica de Chile	Diciembre, 2005	Santiago-Chile
6	1	“Valorización de Instrumentos Financieros en Mercados con Pocas Transacciones: Análisis de una Metodología Basada en un Modelo Dinámico para la Tasa Cero Real en Chile”	JOSE LUIS MANIEU ESPINOSA	13548272-2	Finanzas	Pontificia Universidad Católica de Chile	Agosto, 2005	Santiago-Chile
7	1	“Decisiones de Asset Allocation en Carteras de Inversión de las AFP: Aplicación del Modelo de Black & Litterman”	RODRIGO ALFONSO IBANEZ VILLARROEL		Finanzas	Pontificia Universidad Católica de Chile	Julio, 2005	Santiago-Chile

2.11 DIFUSIÓN Y PUBLICACIONES DE RESULTADOS

PUBLICACIONES RELACIONADAS CON CONTENIDOS DEL PROYECTO.

Nº	**Tipo** 1= libro, 2= cap. de libro 3= art. revista, 4= manuales técnicos, Otros {especificar}	**Nombre Publicación** (del cap. libro, del art. revista, del manual técnico, de otros)	**Nombre** (del libro o revista, cuando en la columna anterior sea un cap. o un art.)	**Nombre autor(es)** **(1)**	**RUT**
1	3	"Term Structure Estimation in Markets with Infrequent Trading“	*Computers & Operations Research*	Cortazar, G., Gravet, M., Urzua, J	60663351-1 9908534-9 13307237-3
2	3	“The Valuation of Multidimensional American Real Options using the LSM Simulation Method”	*International Journal of Finance and Economics*	Cortazar, G, Schwartz, E. S. Naranjo, L	60663351-1 12931431-1
3	3	“An N-Factor Gaussian Model of Oil Futures Prices”	*The Journal of Futures Markets*	Cortazar, G. Naranjo, L.	60663351-1 12931431-1

IDENTIFICACION (CONTINUACION)

Nº **(1)**	**Mes y Año** de Edición o Publicación	**Ciudad y País** (donde se editó o publicó)	**Páginas** (para cap. de libro o art. de revista, de.... a....)	**Editorial**	**Código ISBN, ISSN, ISI (2)**	**Disciplina** (disciplina Fondecyt predominante)	**Clasificación** 1=Científica 2=Tecnológica 3=Difusión Otras {especificar}
1	01/2008	OXFORD, ENGLAND	113 – 129	PERGAMON-ELSEVIER SCIENCE LTD,	ISSN: 0305-0548	Computación	1
2	2007 (por aparecer)	CHICHESTER, ENGLAND, W SUSSEX, PO19 8SQ		JOHN WILEY & SONS INC	ISSN: 1076-9307	Finanzas	1
3	03/2006	HOBOKEN, USA, NJ, 07030	243-268	JOHN WILEY & SONS INC,	ISSN:0270-7314	Finanzas	1

2.12 PROTECCIÓN DE RESULTADOS

a) PATENTES

Nº	Título	**Disciplina** (disciplina Fondecyt predominante)	**País** (país donde se solicitó la patente)	**Páginas** (cantidad de páginas de la patente)	Observaciones	**Estado** (1) Solicitada (2) Otorgada	Fecha Otorgamiento	**Tipo de patente** (Nacional / Internacional) Indique país(es) donde rige
1								

AUTORES (CONTINUACION PATENTES)

Nº patente cuadro anterior	**Rut** (para nacionales y extranjeros residentes en Chile)	Apellido Paterno	Apellido Materno	Nombres	Dueño de la patente

b) OTRAS FORMAS DE PROTECCIÓN

Resultado	Tipo de protección	Establecida *(si o no)*
Resultados del proyecto	El valor comercial de los resultados del proyecto se basan fuertemente en la reputación de objetividad, rigurosidad y compromiso de permanente innovación que puedan comunicar los proveedores de los mismos. Es por esto que más que proteger desarrollos la estrategia de protección consiste en comunicar en forma creíble estos atributos y en ofrecer innovaciones permanentes difíciles de replicar por otros proveedores.	Si

2.13 EVENTOS RELACIONADOS CON EL PROYECTO EN QUE PARTICIPO PERSONAL DEL PROYECTO.

A) IDENTIFICACION DE EVENTOS.

Nº	Título o nombre del evento	**Tipo de Evento** 1=Congreso, 2=Seminario, 3=Taller, 4= Curso, 5=Simposio, 6=Mesa Redonda, Otro {especificar}	**País** (país donde se realizó el evento)	**Ciudad** (ciudad dónde se realizó el evento)	Fecha de inicio del evento	Fecha término del evento	**Clasificación** 1=C&T, 2=de Negocios, 3=de Difusión, 4= de Capacitación, Otros {especificar}	**Evento organizado por** 1= el proyecto Otros {especificar}
1	Seminario Internacional de Innovación Financiera/Lanzamiento RiskAmercaPLUS	2	Chile	Santiago	29-03-2007	29-03-2007	2-3	UC-Fondef
2	4th Annual Conference of Asia Pacific Association of Derivatives	1	India	Gurgaon	20-06-2007	22-06-2007	1	APAD-MDI
3	Latin American Meeting of the Econometric Society	1	México	Ciudad de México	02-11-2006	04-11-2006	1	LAMES-ITAM
4	2006 FMA Annual Meeting	1	EEUU	Salt Lake City	11-10-2006	14-10-2006	1	FMA
5	2006 Far Eastern Meeting of the Econometric Society	1	China	Beijing	9-07-2006	12-07-2006	1	Econometric Society-Tsinghua University
6	INFORMS Hong Kong International 2006	1	China	Hong Kong	25-06-2006	28-06-2006	1	INFORMS
7	15th annual meeting of the European Financial Management Association	1	España	Madrid	28-06-2006	01-07-2006	1	EFMA
8	2005 FMA Annual Meeting	1	EEUU	Chicago	12-10-2005	15-10-2005	1	FMA
9	9th Annual International Conference Real Options: Theory Meets Practice	1	Francia	Paris	23-06-2005	25-06-2005	1	ROG - EDC
10	EFA 2004 Meeting	1	Holanda	Maastricht	18-06-2004	21-08-2004	1	EFA

B) IDENTIFICACION DEL PERSONAL DEL PROYECTO QUE PARTICIPO EN EVENTOS

Nº del evento (Nº de la tabla anterior) (1)	**Descripción de la persona vinculada al evento**				**Rol en el evento** 1= Expositor 2=Asistente Otro {especificar}	**Título de la Exposición** (si fue expositor)	**Certificación** (Si fue asistente) 1= De asistencia 2= De aprobación	**Duración** (si fue asistente) Nº Horas
	RUT	**Apellido paterno**	**Apellido materno**	**Nombres**				
1	6066335-1	Cortazar	Sanz	Gonzalo	1	Lanzamiento Oficial de RiskAmercaPlus		
	4940618-5	Majluf	Sapag	Nicolás	1	Comentarios Finales		
		Schwartz	G.	Eduardo	1	Hedge Funds: Riesgos y Oportunidades		
2	6066335-1	Cortazar	Sanz	Gonzalo	1	“A Multicommodity Model of Futures Prices: Using Futures Prices of One Commodity to Estimate the Stochastic Process of Another”		
3	6066335-1	Cortazar	Sanz	Gonzalo	1	"Term Structure Estimation in Markets with Infrequent Trading"		
4	6066335-1	Cortazar	Sanz	Gonzalo	1	"Term Structure Estimation in Markets with Infrequent Trading"		
5	6066335-1	Cortazar	Sanz	Gonzalo	1	"Term Structure Estimation in Markets with Infrequent Trading"		
6	6066335-1	Cortazar	Sanz	Gonzalo	1	“Using futures prices of one commodity to estimate the stochastic process of another”		
7	6066335-1	Cortazar	Sanz	Gonzalo	1	"Term Structure Estimation in Markets with Infrequent Trading"		
8	6066335-1	Cortazar	Sanz	Gonzalo	1	“An N-Factor Gaussian Model of Oil Futures Prices”		
9	6066335-1	Cortazar	Sanz	Gonzalo	1	“The Valuation of Multidimensional American Real Options using the LSM Simulation Method”		
10	6066335-1	Cortazar	Sanz	Gonzalo	1	"Term Structure Estimation in Low-Frequency Transaction Markets: A Kalman Filter Approach with Incomplete Panel-Data"		

(1) Copie el Nº correspondiente de la tabla anterior

2.14 COOPERACIÓN INTERNACIONAL Y NACIONAL. COLABORACION DE EXPERTOS

Nº	RUT	Apellido Paterno	Apellido Materno	Nombres	Institución de Origen	País de Origen	Lugar de Trabajo en Chile	Fecha de Inicio	Fecha de término	Tema	Disciplina Fondecyt predominante	Código de actividades del proyecto (en las que participó)
1		Schwartz	Greenwald	Eduardo Saul	UCLA	EEUU	En Chile y Via Web	2004	2007	Supervisión Research	Finanzas	Research

3 IMPACTOS ACTUALES Y ESPERADOS DE MEDIANO Y LARGO PLAZO

3.1 IMPACTOS ECONOMICO-SOCIALES.

PRODUCIDOS

Mejor valorización de carteras de inversión para todos los Fondos Mutuos del País y para algunas otras instituciones financieras del país, lo que transparenta los mercados y permite una mejor competencia y asignación de recursos financieros.

Desde marzo de 2006 los precios para todos los instrumentos de renta fija son entregados vía Internet a todos los fondos mutuos nacionales como precio oficial para ser utilizados en el cálculo diario de la cuota de todos los fondos mutuos, según circular de la Superintendencia de Valores y Seguros y en acuerdo con la Asociación de Administradoras de Fondos Mutuos de Chile.

ESPERADOS

Mejores decisiones de inversión y de gestión del riesgo para carteras de inversión de las instituciones financieras del país a medida que vayan adoptando las herramientas que actualmente están en fase de prueba.

3..2 IMPACTOS CIENTIFICO-TECNOLOGICOS.

PRODUCIDOS

Nuevas metodologías de valorización y gestión del riesgo principalmente para mercados con pocas transacciones como son los mercados de economías emergentes como la chilena. Esto se ha traducido en publicaciones, tesis de magíster y presentaciones en conferencias académicas internacionales.

ESPERADOS

Nuevas publicaciones en preparación orientadas a formas de gestionar carteras de inversión y a modelos multi-activos

3..3 IMPACTOS INSTITUCIONALES.

PRODUCIDOS

-Fortalecimiento del FINlabUC-Laboratorio de Investigación Avanzada en Finanzas tanto en actividad, reconocimiento y equipamiento.
-Fortalecimiento del programa de Magíster en Ciencias de la Ingeniería con incremento en el número de alumnos que se especializan en finanzas a nivel de postgrado
-Fortalecimiento de Relaciones con Sector Productivo
-Fortalecimiento de las relaciones de Cooperación Internacional

ESPERADOS

-Incremento en los fortalecimientos institucionales anteriores.

3..4 IMPACTOS AMBIENTALES.

PRODUCIDOS

No existen

ESPERADOS

No existen

3.5 IMPACTOS REGIONALES.

PRODUCIDOS

No existen

ESPERADOS

No existen

4 PLAN DE NEGOCIOS.

A continuación se describen los principales aspectos del Plan de Negocios en ejecución:

4.1 Productos
Se considera la comercialización de 3 módulos de Servicios :
- Módulo **SVC**
- Módulo **Indices**
- Módulo **Portfolio**

4.2 Clientes
Los clientes de los servicios ofrecidos son instituciones financieras (AFP, Bancos, Fodnos Mutuos, Corredoras, Cias de Seguros, etc), y organismos reguladores (Banco Central, Superintendencias, etc).

4.3 Factores de Éxito
El principal Factor de Éxito es la capacidad de generar y comunicar una reputación de objetividad, rigurosidad y compromiso de permanente innovación para los Servicios ofrecidos. Para ello se hace necesario mantener activo un equipo de investigadores haciendo investigación reconocida internacionalmente.

4.4 Comercialización
La comercialización nacional se realizara a través de la plataforma RiskAmerica, la que comunicó al mercado la incorporación de estos servicios expandidos como RiskAmericaPlus. Se están explorando oportunidades de internacionalización de los servicios.

4.5 Ventas Actuales y Proyección de Resultados Futuros
Las ventas esperadas de los 3 módulos de Servicios para el año 2007 alcanzan MM$198 las que debieran incrementarse en los años futuros hasta alcanzar un monto esperado de MM$570 el año 2010.

5. GESTION DEL PROYECTO.

5.1 PLAN DE TRABAJO EFECTIVAMENTE REALIZADO vs. PLAN DE TRABAJO INICIAL.

Comentario:

El proyecto se encuadró bastante bien en el Plan de Trabajo inicial propuesto. Los principales cambios fueron la extensión de la fecha final del proyecto en 1 mes y que algunos de los resultados programados sufrieron un reordenamiento en el tiempo.
La razón principal de estos cambios fueron de acuerdo con cambios en la percepción de los requerimientos de los potenciales clientes.

Enseñanzas obtenidas *¿Qué consideraría para mejorar la formulación y ejecución de un próximo plan?*

5.2 GASTO EJECUTADO vs. PRESUPUESTO INICIAL.

Comentario

El proyecto se ajustó bastante bien al presupuesto inicial total presentado, con pequeños ajustes en algunos ítems.

Enseñanza obtenida *¿Qué consideraría para la formulación y ejecución de un próximo presupuesto?*

5.3 INSTITUCIONES PARTICIPANTES

A.- AL INICIO DEL PROYECTO

INSTITUCION	**ROL**	**APORTES** *(comprometido en contrato CONCYT, pesos de 1996)*
A.F.P. Habitat		**103,500**
Riskamerica/Dictuc SA		**125,000**
P.U.C.		**122,850**

B.-DURANTE EL PROYECTO

INSTITUCION	**ROL**	**APORTES** *(reales efectuados en moneda del año)*	**OBSERVACIONES** *(causas o motivo del retiro, incorporación o modificación del aporte)*
A.F.P. Habitat		**104,912**	
Riskamerica/Dictuc SA		**125,001**	
P.U.C.		**122,873**	

C.- ENSEÑANZA OBTENIDA

5.4 RENDICION FINAL DE GASTOS.

INSTITUCIÓN BENEFICIARIA	**MONTO TOTAL RENDIDO** *(aprobado)* **(1)**	**MONTO TOTAL CONTRATO** *(adjudicado mas reajustes)* **(2)**	**MONTO TOTAL GIRADO POR FONDEF (3)**	**MONTO DEVOLUCIÓN O GIRO (3)-(1) ***
Pont. Universidad Católica de Chile	*153.632.760*	*158.000.000*	*155.303.294*	*- 1.670.534*

5.5 ORGANIZACIÓN Y EQUIPO DE TRABAJO

El proyecto se organizó considerando:

Una dirección General: Gonzalo Cortazar

Una unidad de Investigación: Eduardo Schwartz y Gonzalo Cortazar (con apoyo de Jaime Casassus)

Una unidad de Productos y Transferencia a Usuarios: Nicolás Majluf y C Mery (AFP Habitat).

Un pool de profesionales e investigadores que eran asignados a distintos proyecto de acuerdo a las necesidades

Las tareas se gestionaron considerando:

Subproyectos a cargo de distintos profesionales, con reportes semanales internos, reportes mensuales con reunión directiva ampliada (con participación de prof AFP) y reuniones trimestrales del directorio del proyecto que incluía al Gte de Inversiones de la AFP.

Enseñanza obtenida:

5.6 OBSERVACIONES, CONCLUSIONES Y RECOMENDACIONES DE LA DIRECCIÓN DEL PROYECTO

1.-Sobre el proyecto

2.- Sobre su Institución

3.- Sobre otras instituciones y empresas patrocinantes

4.- Sobre el FONDEF

5.7 OBSERVACIONES, CONCLUSIONES Y RECOMENDACIONES INSTITUCIONALES

GOBIERNO DE CHILE
CONICYT

IV PARTE. INFORME CIENTÍFICO TECNOLÓGICO

Y ECONÓMICO SOCIAL

Código Proyecto: D03I1039

Nombre del Proyecto: DESARROLLO DE HERRAMIENTAS COMPUTACIONALES PARA OPTIMIZAR LA GESTION DE CARTERAS DE INVERSION EN MERCADOS EMERGENTES: APLICACION A LOS FONDOS DE PENSIONES EN CHILE

Instituciones Participantes: Pontificia Universidad Católica de Chile

Otros Participantes: AFP Habitat S.A.
RiskAmerica- Dictuc S.A.

Director del Proyecto: .Gonzalo Cortazar Sanz, Firma:.........

Fecha de emisión : 23/07/2007

FONDEF
FOMENTO AL DESARROLLO CIENTIFICO Y TECNOLOGICO

IV PARTE. INFORME CIENTÍFICO TECNOLÓGICO Y ECONÓMICO SOCIAL

1. INDICE

1.1 Índice por Tema de Investigación

1: Modelación y Calibración de Procesos Estocásticos para Precios de Instrumentos en Mercados con Paneles de Datos Incompletos.

2: Metodologías de Valorización de Derivados escritos sobre Subyacentes con procesos Completos

3: Metodologías de Medición de Spreads.

4: Metodologías de Medición de Riesgos y de Asignación de Activos (*Asset Allocation*) para Carteras de Inversión

5: Modelación y Calibración Conjunta de Procesos Estocásticos de Múltiples Activos.

1.2 Índice por Documento de Resultados

1.2.1 Publicaciones

Cortazar, G., Gravet, M., Urzua, J. (2008) "The Valuation of Multidimensional American Real Options using the LSM Simulation Method" *Computers & Operations Research* Vol 35 (2008) 113 – 129

Cortazar, G, Schwartz, E. S., Naranjo, L, (2007) "Term Structure Estimation in Markets with Infrequent Trading" *International Journal of Finance and Economics* (por aparecer)

Cortazar, G., Naranjo, L. (2006) "An N-Factor Gaussian Model of Oil Futures Prices" *The Journal of Futures Markets*, Vol.26, No. 3, March, 2006, 243-268

1.2.2 Documentos de Trabajo aún no publicados

Cortazar, G, Milla, C. Severino, F. (2007) "A Multicommodity Model of Futures Prices: Using Futures Prices of One Commodity to Estimate the Stochastic Process of Another"

Cortazar, G, Bernales, A. Beuermann, D. (2007) "Methodology and Implementation of Value-at-Risk Measures in Emerging Fixed-Income Markets with Infrequent Trading

1.2.3 Tesis de Magíster Finalizadas

"Modelo estocástico multicommodity para la dinámica de precios de contratos futuros. Selección y estimación del modelo utilizando componentes principales comunes y filtro de Kalman", FELIPE SEVERINO DIAZ, Tesis de Magíster en Ciencias de la Ingeniería, Pontificia Universidad Católica de Chile, 26-03-2007

"Estimación de Spreads por Liquidez en un Mercado con Pocas Transacciones: El Caso del Mercado de Bonos del Banco Central de Chile", PEDRO MATÍAS MORAL MESA, Tesis de Magíster en Ciencias de la Ingeniería, Pontificia Universidad Católica de Chile, 17-01-2006

“Metodología e Implementación de Métodos de VALUE AT RISK en Mercados de Renta Fija con baja Frecuencia de Transacciones” ALEJANDRO ADRIAN BERNALES SILVA, Tesis de Magíster en Ciencias de la Ingeniería, Pontificia Universidad Católica de Chile, 23-12-2005

“Modelos Estocásticos de Precios de Commodities y Estimación Conjunta de la Dinámica de dos Commodities Mediante el Filtro de Kalman” CARLOS IGNACIO MILLA GONZALEZ, Tesis de Magíster en Ciencias de la Ingeniería, Pontificia Universidad Católica de Chile, 23-12-2005

1.2.4 Memorias de Título Finalizadas

“Bonos Corporativos: una Revisión del Mercado y una Aproximación a un Método Práctico de Valorización” , Memoria Escuela de Ingeniería, Pontificia Universidad Católica de Chile, CLAUDIO EDUARDO HELFMANN SOTO , 31-12-2005

“Valorización de Instrumentos Financieros en Mercados con Pocas Transacciones: Análisis de una Metodología Basada en un Modelo Dinámico para la Tasa Cero Real en Chile” JOSE LUIS MANIEU ESPINOSA, Memoria Escuela de Ingeniería, Pontificia Universidad Católica de Chile 12-08-2005

“Decisiones de Asset Allocation en Carteras de Inversión de las AFP: Aplicación del Modelo de Black & Litterman”, RODRIGO ALFONSO IBANEZ VILLARROEL, Memoria Escuela de Ingeniería, Pontificia Universidad Católica de Chile, 26-07-2005

1.2.5 Presentaciones en Congresos Académicos

Cortazar, G, Milla, C. Severino, F. (2007) “A Multicommodity Model of Futures Prices: Using Futures Prices of One Commodity to Estimate the Stochastic Process of Another”,4th Annual Conference of Asia Pacific Association of Derivatives (APAD), Gurgaon, India, June 20-22, 2007

Cortazar, G, Schwartz, E. S., Naranjo, L, (2006) "Term Structure Estimation in Markets with Infrequent Trading". Latin American Meeting of the Econometric Society (LAMES) , ITAM, Ciudad de México, Nov. 2-4, 2006

Cortazar, G, Schwartz, E. S., Naranjo, L, (2006) "Term Structure Estimation in Markets with Infrequent Trading". 2006 FMA Annual Meeting , Salt Lake City, Oct. 11 - 14, 2006

Cortazar, G, Schwartz, E. S., Naranjo, L, (2006) "Term Structure Estimation in Markets with Infrequent Trading". 2006 Far Eastern Meeting of the Econometric Society, Beijing, July 9-12, 2006

Cortazar, G, Milla, C. Severino, F. (2006) “Using futures prices of one commodity to estimate the stochastic process of another” INFORMS Hong Kong International 2006, Hong Kong, June 25-28, 2006

Cortazar, G, Schwartz, E. S., Naranjo, L, (2006) "Term Structure Estimation in Markets with Infrequent Trading". 15th annual meeting of the European Financial Management Association, Madrid, June 28-July 1, 2006

Cortazar, G., Naranjo, L. (2005) “An N-Factor Gaussian Model of Oil Futures Prices” 2005 FMA Annual Meeting, Chicago, October 12-15, http://www.fma.org/Chicago/ChicagoProgram.htm

Cortazar, G., Gravet, M., Urzua, J. (2005) “The Valuation of Multidimensional American Real Options using the LSM Simulation Method”, 9th Annual International Conference Real Options: Theory Meets Practice, Real Options Group and EDC Paris, Paris, June 23-25, http://www.realoptions.org/AcademicProgram/academicprogram2005.html

Cortazar, G, Schwartz, E. S., Naranjo, L, (2004) "Term Structure Estimation in Low-Frequency Transaction Markets: A Kalman Filter Approach with Incomplete Panel-Data" (March 2004). EFA 2004 Maastricht Meetings Paper No. 3102. , Maastricht, August 18-21, http://ssrn.com/abstract=567090

2. INVESTIGACIÓN Y DESARROLLO REALIZADA POR EL PROYECTO

Durante el desarrollo del proyecto se realizaron múltiples investigaciones científicas, las que fueron luego incorporadas a diversas aplicaciones computacionales.

En lo que sigue se enumera una serie de **Temas** o **Problemáticas** que dieron origen a resultados científicos originales que quedaron documentados. Para cada Tema se incluye una breve descripción del problema y resultado y cómo éstos quedaron documentados en términos de Publicaciones, Documentos de trabajo, Tesis de Magíster, Memorias de Título y/o Presentaciones en Congresos Académicos Internacionales.

Tema 1: Modelación y Calibración de Procesos Estocásticos para Precios de Instrumentos en Mercados con Paneles de Datos Incompletos.

Discusión:

Se perfecciona metodología cuya investigación se inició en fecha anterior al inicio del proyecto (financiado parcialmente por proyecto Fondef D00I1024) orientado a cómo determinar la mejor curva de precios de hoy (y su dinámica a través del tiempo) en presencia de un número limitado de precios producto de la falta de liquidez del mercado.
La estrategia propuesta consiste en calibrar un modelo multifactorial para la dinámica de los precios y utilizar filtros de Kalman calibrados con paneles incompletos.

Publicaciones

Cortazar, G, Schwartz, E. S., Naranjo, L, (2007) "Term Structure Estimation in Markets with Infrequent Trading“ *International Journal of Finance and Economics* (por aparecer)

Cortazar, G., Naranjo, L. (2006) “An N-Factor Gaussian Model of Oil Futures Prices” *The Journal of Futures Markets*, Vol.26, No. 3, March, 2006, 243-268

Presentaciones en Congresos Académicos

Cortazar, G, Schwartz, E. S., Naranjo, L, (2006) "Term Structure Estimation in Markets with Infrequent Trading". Latin American Meeting of the Econometric Society (LAMES) , ITAM, Ciudad de México, Nov. 2-4, 2006

Cortazar, G, Schwartz, E. S., Naranjo, L, (2006) "Term Structure Estimation in Markets with Infrequent Trading". 2006 FMA Annual Meeting , Salt Lake City, Oct. 11 - 14, 2006

Cortazar, G, Schwartz, E. S., Naranjo, L, (2006) "Term Structure Estimation in Markets with Infrequent Trading". 2006 Far Eastern Meeting of the Econometric Society, Beijing, July 9-12, 2006

Cortazar, G, Schwartz, E. S., Naranjo, L, (2006) "Term Structure Estimation in Markets with Infrequent Trading". 15th annual meeting of the European Financial Management Association, Madrid, June 28-July 1, 2006

Cortazar, G., Naranjo, L. (2005) “An N-Factor Gaussian Model of Oil Futures Prices” 2005 FMA Annual Meeting, Chicago, October 12-15, http://www.fma.org/Chicago/ChicagoProgram.htm

Cortazar, G, Schwartz, E. S., Naranjo, L, (2004) "Term Structure Estimation in Low-Frequency Transaction Markets: A Kalman Filter Approach with Incomplete Panel-Data" (March 2004). EFA 2004 Maastricht Meetings Paper No. 3102. , Maastricht, August 18-21, http://ssrn.com/abstract=567090

Tema 2: Metodologías de Valorización de Derivados escritos sobre Subyacentes con procesos Completos

Discusión:

Aún cuando existen múltiples procedimientos para valorizar opciones de tipo Americano (cuya estrategia óptima de ejercicio no es evidente), la complejidad para su implementación crece exponencialmente con la dimensión del problema a resolver. Dado que los modelos de precios actuales requeridos para representar adecuadamente la dinámica de tasas de interés y de precios son multifactoriales, estas metodologías tradicionales son en la práctica inutilizables para valorizar los activos derivados escritos sobre estas tasas de interés o precios.
La estrategia de resolución propuesta consiste en adaptar metodologías recientes que usan un método que combina simulación de Montecarlo (forward) con resolución backward de árboles construidos a partir de estas simulaciones.

Publicaciones

Cortazar, G., Gravet, M., Urzua, J. (2008) "The Valuation of Multidimensional American Real Options using the LSM Simulation Method" *Computers & Operations Research* Vol 35 (2008) 113 – 129

Presentaciones en Congresos Académicos

Cortazar, G., Gravet, M., Urzua, J. (2005) "The Valuation of Multidimensional American Real Options using the LSM Simulation Method", 9th Annual International Conference Real Options: Theory Meets Practice, Real Options Group and EDC Paris, Paris, June 23-25, http://www.realoptions.org/AcademicProgram/academicprogram2005.html

Tema 3: Metodologías de Medición de Spreads.

Discusión:
Existen diversos tipos de spreads o diferenciales de precios (o tasas) entre los activos más deseados por el mercado (los que se transan a mayores precios, o equivalentemente descontados a las menores tasas) y el resto. Este diferencial se puede deber a la existencia de riesgos de crédito o de liquidez que explican que inversionistas racionales sólo los adquieren en la medida que se transen a un descuento relativo los "mejores" activos del mercado.
Durante la realización del proyecto se desarrollaron metodologías de estimación de spreads para diversos instrumentos de deuda (bonos de empresas, letras hipotecarias, depósitos a plazo, Bonos de Reconocimiento, etc) así como estimaciones de los spreads de liquidez presentes entre activos de la misma familia (Bonos Banco Central) pero que exhiben distinta liquidez (ej PRC 8 años versus PRC a 10 años).

Tesis de Magíster

"Estimación de Spreads por Liquidez en un Mercado con Pocas Transacciones: El Caso del Mercado de Bonos del Banco Central de Chile", PEDRO MATÍAS MORAL MESA, Tesis de Magíster en Ciencias de la Ingeniería, Pontificia Universidad Católica de Chile, 17-01-2006

Memorias de Título

"Bonos Corporativos: una Revisión del Mercado y una Aproximación a un Método Práctico de Valorización" , Memoria Escuela de Ingeniería, Pontificia Universidad Católica de Chile, CLAUDIO EDUARDO HELFMANN SOTO , 31-12-2005

"Valorización de Instrumentos Financieros en Mercados con Pocas Transacciones: Análisis de una Metodología Basada en un Modelo Dinámico para la Tasa Cero Real en Chile" JOSE LUIS MANIEU ESPINOSA, Memoria Escuela de Ingeniería, Pontificia Universidad Católica de Chile 12-08-2005

Tema 4: Metodologías de Medición de Riesgos y de Asignación de Activos (*Asset Allocation*) para Carteras de Inversión

Existe una extensa literatura de cómo medir los riesgos financieros en una cartera e inversión (Value at Riks, Tracking error, etc) y de cómo tomar decisiones de Asignación de Activos (Asset Allocation) que permitan mejorar el proceso de inversiones.
Sin embargo, para poder resolver estos problemas en mercados emergentes como el chileno, con ausencia de transacciones hace falta modificar procedimientos y generar información confiable relativa al comportamiento de las distintas clases de activos, entre otros aspectos.

Documentos de Trabajo aún no publicados

Cortazar, G, Bernales, A. Beuermann, D. (2007) "Methodology and Implementation of Value-at-Risk Measures in Emerging Fixed-Income Markets with Infrequent Trading

Tesis de Magíster

"Estimación de Spreads por Liquidez en un Mercado con Pocas Transacciones: El Caso del Mercado de Bonos del Banco Central de Chile", PEDRO MATÍAS MORAL MESA, Tesis de Magíster en Ciencias de la Ingeniería, Pontificia Universidad Católica de Chile, 17-01-2006

"Metodología e Implementación de Métodos de VALUE AT RISK en Mercados de Renta Fija con baja Frecuencia de Transacciones" ALEJANDRO ADRIAN BERNALES SILVA, Tesis de Magíster en Ciencias de la Ingeniería, Pontificia Universidad Católica de Chile, 23-12-2005

Memorias de Título

"Decisiones de Asset Allocation en Carteras de Inversión de las AFP: Aplicación del Modelo de Black & Litterman", RODRIGO ALFONSO IBANEZ VILLARROEL, Memoria Escuela de Ingeniería, Pontificia Universidad Católica de Chile, 26-07-2005

Tema 5: Modelación y Calibración Conjunta de Procesos Estocásticos de Múltiples Activos.

Durante el desarrollo del proyecto se hizo evidente que en algunas situaciones se hace conveniente utilizar información de precios de ciertos instrumentos financieros para estimar de mejor manera el precio de otro instrumento que no fue transado, pero que históricamente ha exhibido retornos correlacionados parcialmente entré sí.
El inicio de esta línea de investigación está siendo muy prometedora permitiendo mejorar sustancialmente los modelos para la dinámica de precios que inicialmente consideraban sólo una familia de instrumentos.

Documentos de Trabajo aún no publicados

Cortazar, G, Milla, C. Severino, F. (2007) "A Multicommodity Model of Futures Prices: Using Futures Prices of One Commodity to Estimate the Stochastic Process of Another"

Tesis de Magíster

"Modelo estocástico multicommodity para la dinámica de precios de contratos futuros. Selección y estimación del modelo utilizando componentes principales comunes y filtro de Kalman", FELIPE SEVERINO DIAZ, Tesis de Magíster en Ciencias de la Ingeniería, Pontificia Universidad Católica de Chile, 26-03-2007
"Estimación de Spreads por Liquidez en un Mercado con Pocas Transacciones: El Caso del Mercado de Bonos del Banco Central de Chile", PEDRO MATÍAS MORAL MESA, Tesis de Magíster en Ciencias de la Ingeniería, Pontificia Universidad Católica de Chile, 17-01-2006

"Modelos Estocásticos de Precios de Commodities y Estimación Conjunta de la Dinámica de dos Commodities Mediante el Filtro de Kalman" CARLOS IGNACIO MILLA GONZALEZ, Tesis de Magíster en Ciencias de la Ingeniería, Pontificia Universidad Católica de Chile, 23-12-2005

Presentaciones en Congresos Académicos

Cortazar, G, Milla, C. Severino, F. (2007) "A Multicommodity Model of Futures Prices: Using Futures Prices of One Commodity to Estimate the Stochastic Process of Another",4th Annual Conference of Asia Pacific Association of Derivatives (APAD), Gurgaon, India, June 20-22, 2007

3. OTROS INFORMES TÉCNICOS

4. EVALUACIÓN CIENTÍFICO-TECNOLÓGICA

A continuación se resume un análisis FODA del proyecto

<u>Fortalezas del Proyecto</u>

-Las metodologías científicas desarrolladas para abordar ausencia de transacciones

-La plataforma tecnológica-computacional incluyendo rutinas computacionales y plataforma WEB.

-Las bases de datos construidas

-El equipo humano especializado capacitado

-La reputación en el mercado

<u>Debilidades del Proyecto</u>

-La vulnerabilidad financiera que lo expone a ataques de eventuales competidores que inicien una guerra de precios.

-Exigencia de mantener innovación permanente como protección de mercado.

<u>Oportunidades del Proyecto</u>

-Posibilidades de expansión internacional.

<u>Amenazas</u>

-Entrada de competencia nacional e internacional.

5. EVALUACIÓN ECONÓMICO-SOCIAL

Análisis comparativo con la evaluación ex-ante de la Formulación del Proyecto.

A continuación se discute y actualiza la evaluación económico social presentada en la Formulación del Proyecto.

Tal como se plantea en la Formulación del proyecto, el beneficio social principal de este proyecto se genera al contribuir a la modernización del mercado financiero nacional. Un resultado exitoso en cuanto a nuevas herramientas de gestión de carteras, como es el caso de lo ocurrido con este proyecto debiera inducir que las administradoras de fondos de pensiones, pueden mejorar la gestión de sus carteras y de esta manera obtener mayores retornos de sus inversiones, sin incrementar el riesgo asumido.

El principal impacto económico social del proyecto es el **Incremento Marginal de la Rentabilidad** aplicado a una **Fracción** de los **Fondos Potenciales** que pudieran verse beneficiados con las herramientas de gestión de carteras desarrolladas. Por último, se debe estimar el **adelantamiento** de los flujo**s** (en número de años) que representa la realización del proyecto comparado con la situación base sin proyecto (i.e. se estima que si no hubiera habido proyecto, otro similar se hubiera desarrollado teniendo el mismo impacto después).

Los cuatro parámetros anteriores son difíciles de estimar y de ellos depende el resultado de los indicadores económicos-sociales.

En la **Evaluación ex–ante** presentada en la formulación del proyecto se asumieron los siguientes valores para estos parámetros:

Incremento Marginal de la Rentabilidad = 0,25%

Fracción = 25% de las administradoras* 60% de los activos = 15%

Fondos Potenciales = MMUS$ 40.000

Adelantamiento = 1 año (si no se hace el proyecto los flujos se realizan 1 año después).

Con lo que el beneficio económico-social por año se estimó en US$ 15 millones y los flujos netos del proyecto presentados en su Formulación bajo estos supuestos fueron:

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FLUJO NETO

Ingresos	0	0	10.754	238	238	119	119	0	0	0
Costos	0	0	276	136	136	70	71	38	38	0
Inversión	0	-509	-509	-6	-6	-6	-6	-6	-6	-6
Costo total I+D	290	219	0	0						
Fondef	97	97	0	0						
Beneficio neto	-290	290	10.987	108	108	55	54	-32	-32	6

TIR	**568,06%**
VAN (10%) MM$	**9.244**
VAN/VAI	**18,90**
VAN/FONDEF	**50,14**

Nota: M/N = moneda nacional
M/E = moneda extranjera

Como se planteó anteriormente, la estimación de los parámetros anteriores está sujeta a bastante incertidumbre. Sin embargo se puede señalar que desde la formulación del proyecto uno de los parámetros se ha incrementado significativamente, al aumentar el parámetro **Fondos Potenciales** subió al año 2007 a más de **MM US$100.000**, es decir **2,5** veces mayor que el valor estimado en la Formulación. Esto permitiría dividir por 2,5 alguno de los indicadores anteriores (por ejemplo suponer que el **Incremento Marginal de la Rentabilidad** fuera 0,1 en vez de los 0,25 asumidos originalmente) y conservar el valor de los indicadores originales.

Debido a lo anterior, una evaluación conservadora mantendría los indicadores económico-sociales presentados en la Formulación inicial.

DESCRIPCION DE LA SITUACION SIN PROYECTO

Para analizar la situación sin proyecto, se ha hecho el supuesto que en caso de no realizarse este proyecto, otro con similares objetivos, pero sin algunas de las sinergias presentes en este proyecto, se desarrollaría con un retraso de sólo un año. Este supuesto de evaluación busca reflejar la dinámica actual de modernización financiera que está teniendo el mercado, que si bien está convergiendo a los estándares de mercados desarrollados, en algunas áreas como las que aborda este proyecto no lo hace con suficiente velocidad.

No se está condicionando la modernización del sistema chileno a este proyecto, sino que se está suponiendo que al hacerlo se adelantan los resultados con el consecuente beneficio para los usuarios.

La evaluación considera que tanto los ingresos como los costos obtenidos por este desarrollo privado son equivalentes a la situación con proyecto, pero que se obtienen un año después. Además la inversión inicial es superior, ya que para obtener los sistemas adecuados, estos deben ser adquiridos y desarrollados por consultores externos en el extranjero quienes deben estudiar el comportamiento del mercado nacional para luego desarrollar desde cero los productos.

DESCRIPCION DE LA SITUACION CON PROYECTO

La situación con proyecto considera que el proyecto adelanta los beneficios económicos-sociales y que el costo de desarrollo es menor en este proyecto que en su eventual competencia.

MEMORIA DE CÁLCULO DE LA EVALUACIÓN ECONÓMICO-SOCIAL

SIN PROYECTO

(La información aquí contenida se extrae de la hoja "SITUACION SIN PYTO")

En millones de pesos

Variables Críticas	Var. 1	Var. 2	Var. 3
Unidad (m3, kg, l, ton, etc.)	%	%	%
Valor Escenario Pesimista	0,10%	20%	50%
Valor Escenario Optimista	0,50%	50%	100%
Valor Más Probable	0,25%	25%	60%

Descripción Variable 1: Porcentaje de Rentabilidad Adicional Anual de Fondos de Pensiones

Descripción Variable 2: Porcentaje de Cartera Administrada que es impactada por nuevas tecnologías de Gestión Optima de Cateras

Descripción Variable 3: Porcentaje de Activos en Cartera que es impactada por nuevas tecnologías de Gestión Optima de Cateras

AÑO	1	2	3	4	5	6	7	8	9	10
INGRESOS										
Moneda nacional	0	0	0	297	535	772	891	1010	1069	1129
Moneda extranjera equivalente	0	0	0	10457	10457	10457	10457	10457	10457	10457
Total ingresos	**0**	**0**	**0**	**10754**	**10991**	**11229**	**11348**	**11466**	**11526**	**11585**
COSTOS										
Mano de obra calificada	0	0	0	226	357	487	552	616	646	676
Mano de obra no calificada	0	0	0	0	0	0	0	0	0	0
Insumos (M/N)	0	0	0	24	26	29	32	35	39	43
Bienes de capital (M/N)	0	0	0	14	15	17	19	20	23	25
Otros (M/N)	0	0	0	12	13	15	16	18	19	21
Total costos	**0**	**0**	**0**	**276**	**412**	**548**	**618**	**689**	**727**	**765**
INVERSIONES										
En moneda nacional	0	509	509	6	6	6	6	6	6	6
En moneda extranjera	0	0	0	0	0	0	0	0	0	0
Total inversiones	**0**	**509**	**509**	**6**	**6**	**6**	**6**	**6**	**6**	**6**
BENEFICIOS	**0**	**-509**	**-509**	**10472**	**10573**	**10675**	**10723**	**10772**	**10793**	**10815**

CON PROYECTO

(La información aquí contenida se extrae de la hoja "SITUACION CON PYTO")

En millones de pesos

Variables Críticas	**Var. 1**	**Var. 2**	**Var. 3**
Unidad (m3, kg, l, ton, etc.)	**%**	**%**	**%**
Valor Escenario Pesimista	0,10%	20%	50%
Valor Escenario Optimista	0,50%	50%	100%
Valor Más Probable	0,25%	25%	60%

Descripción Variable 1:	Porcentaje de Rentabilidad Adicional Anual de Fondos de Pensiones
Descripción Variable 2:	Porcentaje de Cartera Administrada que es impactada por nuevas tecnologías de Gestión Optima de Cateras
Descripción Variable 3:	Porcentaje de Activos en Cartera que es impactada por nuevas tecnologías de Gestión Optima de Cateras

AÑO	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
INGRESOS										
Ingresos por ventas	0	0	297	535	772	891	1010	1010	1069	1129
Externalidad Positiva 1	0	0	10457	10457	10457	10457	10457	10457	10457	10457
Total ingresos	**0**	**0**	**10.754**	**10.991**	**11.229**	**11.348**	**11.466**	**11.466**	**11.526**	**11.585**
COSTOS										
Mano de obra calificada	0	0	226	357	487	552	616	646	676	676
Mano de obra no calificada	0	0	0	0	0	0	0	0	0	0
Insumos (M/N)	0	0	24	26	29	32	35	39	43	43
Bienes de capital (M/N)	0	0	14	15	17	19	20	23	25	25
Otros (M/N)	0	0	12	13	15	16	18	19	21	21
Total costos	**0**	**0**	**276**	**412**	**548**	**618**	**689**	**727**	**765**	**765**
INVERSIONES										
En moneda nacional	0	0	0	0	0	0	0	0	0	0
En moneda extranjera	0	0	0	0	0	0	0	0	0	0
Total inversiones	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
COSTO I&D (parciales por año)										
En M/N	290	219	0	0						
En M/E	0	0	0	0						
Total I&D (todos los aportes)	**290**	**219**	**0**	**0**						
Sólo lo solicitado a FONDEF	97	97	0	0						
BENEFICIOS	-290	-219	10.478	10.579	10.681	10.729	10.778	10.740	10.761	10.821

FLUJO NETO

Ingresos	0	0	10.754	238	238	119	119	0	0	0
Costos	0	0	276	136	136	70	71	38	38	0
Inversión	0	-509	-509	-6	-6	-6	-6	-6	-6	-6
Costo total I+D	290	219	0	0						
Fondef	97	97	0	0						
Beneficio neto	-290	290	10.987	108	108	55	54	-32	-32	6

TIR	**568,06%**
VAN (10%) MM$	**9.244**
VAN/VAI	**18,90**
VAN/FONDEF	**50,14**

Nota: M/N = moneda nacional
M/E = moneda extranjera

SITUACION SIN PROYECTO
(Situación para primer año de actividades)

MERCADO SITUACION SIN PROYECTO (Para cada Producto o Servicio a evaluar)

AÑO	0	1	2	3	4	5	6	7
Mercado Sin Proyecto. Producto o Sevicio 1 (Unidades)	0	0	0	0	5	9	13	15

AÑO	8	9	10
Mercado Sin Proyecto. Producto o Servicio 1 (Unidades)	17	18	19

INGRESOS

Moneda	Pesos	Tipo de cambio $	$697

Productos o servicios	Tipo de unidad a considerar	Precio Unitario ($)	Cantidad anual Mercado Nacional (Unidades)	Cantidad anual Mercado Export. (Unidades)	Ingresos Mercado Nacional (MM$)	Ingresos Mercado Exportador (MM$)	Total de Ingresos (MM$)
1	Servicios a Usuarios	59400000	5	0	297	0	$297
2		0	0	0	0	0	$0
n		0	0	0	0	0	$0
TOTAL INGRESOS			**5**	**0**	**297**	**0**	**297**

COSTOS

MANO DE OBRA CALIFICADA

Nº Puestos	Cargos	Costo mensual unitario $	Costo total mensual (MM$)	Costo total anual (MM$)
1	Gerente	4000000	4	48
7	Profesional	1350000	9,45	113,4
5	Tecnico	750000	3,75	45
3	Administrativo	550000	1,65	19,8
0	Otros (definir)	0	0	0
	TOTALES		**18,85**	**226,2**

MANO DE OBRA NO CALIFICADA

Nº Puestos	Cargos	Costo mensual unitario $	Costo total mensual (MM$)	Costo total anual (MM$)
0	Obreros y Jornaleros	0	0	0
0	Otros (definir)	0	0	0
	TOTALES		**0**	**0**

INSUMOS PARA LA PRODUCCION

Insumos para la producción	Tipo de unidad a considerar	Costo Unitario ($)	Cantidad mensual (Unidades)	Costo total mensual (MM$)	Costo total anual (MM$)
1	Insumos de Oficina Promedio	500000	4	2	24
2		0	0	0	0
3		0	0	0	0
	TOTALES			**2**	**24**

BIENES DE CAPITAL

Moneda | **Tipo de cambio $**

Bienes de Capital	Tipo de unidad a considerar	Cantidad	Costo Unitario ($)	Costo total anual (MM$)
1	N° computadores	8	500000	4
2	Mantención Oficinas	10	1000000	10
3		0	0	0
	TOTALES			**14**

OTROS COSTOS

Otros costos	Tipo de unidad a considerar	Costo Unitario ($)	Cantidad mensual (Unidades)	Costo total mensual (MM$)	Costo total anual (MM$)
1	Arriendo Mensual	1000000	1	1	12
2		0	0	0	0
3		0	0	0	0
	TOTALES			**1**	**12**

TOTAL COSTOS ANUALES	**$276**

RESUMEN DE INVERSIONES

ITEM	EN MONEDA NACIONAL (MM$)	EN MONEDA EXTRANJERA (MM$)	TOTAL GENERAL (MM$)
Terreno	0	0	0
Infraestructura física	0	0	0
Maquinaria y equipos	0	0	0
Otros (definir)	1018	0	1018
TOTAL	**1018**	**0**	**1018**

SITUACION CON PROYECTO
(Situación para primer año de actividades)

CURVA DE ADOPCION DE LA TECNOLOGIA (Para cada Producto o Servicio)

AÑO	0	1	2	3	4	5	6	7
Mercado Con Proyecto Producto 1 (Unidades)	0	0	0	5	9	13	15	17

AÑO	8	9	10
Mercado Con Proyecto Producto 1 (Unidades)	17	18	19

INGRESOS

Moneda Extranjera		Tipo de cambio $	

Productos o servicios	Tipo de unidad a considerar	Precio Unitario ($)	Cantidad anual Mercado Nacional (Unidades)	Cantidad anual Mercado Export. (Unidades)	Ingresos Moneda Nacional (MM$)	Ingresos Moneda Extranjera (MM$)	Total de Ingresos (MM$)
1	Servicios a Usuarios	59400000	5	0	297	0	$297
2		0	0	0	0	0	$0
n		0	0	0	0	0	$0
	TOTAL		**5**	**0**	**297**	**0**	**297**

COSTOS

MANO DE OBRA CALIFICADA

Moneda Extranjera		Tipo de cambio $	

(Para todos los productos y todas las Unidades de Negocio)

Nº Puestos	Cargos	Costo mensual unitario $	Costo total mensual (MM$)	Costo total anual Moneda Nacional (MM$)	Costo total anual Moneda Extranjera (MM$)
1	Gerente	4000000	4	48	0
7	Profesional	1350000	9,45	113,4	0
5	Tecnico	750000	3,75	45	0
3	Administrativo	550000	1,65	19,8	0
0	Otros (definir)	0	0	0	0
	TOTALES		**18,85**	**226,2**	**0**

MANO DE OBRA NO CALIFICADA

Moneda Extranjera		Tipo de cambio $	

(Para todos los productos y todas las Unidades de Negocio)

Nº Puestos	Cargos	Costo mensual unitario $	Costo total mensual (MM$)	Costo total anual Moneda Nacional (MM$)	Costo total anual Moneda Extranjera (MM$)
0	Obreros y Jornaleros	0	0	0	0
0	Otros (definir)	0	0	0	0
	TOTALES		**0**	**0**	**0**

INSUMOS PARA LA PRODUCCION

Moneda Extranjera | Tipo de cambio $

(Para todos los productos y todas las Unidades de Negocio)

Insumos para la producción	Tipo de unidad a considerar	Costo Unitario ($)	Cantidad mensual (Unidades)	Costo total mensual (MM$)	Costo total anual Moneda Nacional (MM$)	Costo total anual Moneda Extranjera (MM$)
1	Insumos de Oficina Promedio	500000	4	2	24	0
2		0	0	0	0	0
3		0	0	0	0	0
	TOTALES			**2**	**24**	**0**

BIENES DE CAPITAL

Moneda Extranjera | Tipo de cambio $

(Para todos los Productos y todas las Unidades de Negocio)

Bienes de Capital	Tipo de unidad a considerar	Cantidad	Costo Unitario ($)	Costo total anual Moneda Nacional (MM$)	Costo total anual Moneda extranjera (MM$)
1	N° computadores	8	500000	4	0
2	Mantención Oficinas	10	1000000	10	0
3		0	0	0	0
	TOTALES			**14**	**0**

OTROS COSTOS

Moneda Extranjera | Tipo de cambio $

(Para todos los Productos y todas las Unidades de Negocio)

Otros costos	Tipo de unidad a considerar	Cantidad	Costo Unitario $	Costo total anual Moneda Nacional (MM$)	Costo total anual Moneda Extranjera (MM$)
1	Arriendo Mensual	12	1000000	12	0
2		0	0	0	0
3		0	0	0	0
	TOTALES			**12**	**0**

TOTAL COSTOS ANUALES MONEDA NACIONAL (MM$)	**$276**

RESUMEN DE INVERSIONES

ITEM	EN MONEDA NACIONAL (MM$)	EN MONEDA EXTRANJERA (MM$)	TOTAL GENERAL (MM$)
Terreno	0	0	0
Infraestructura física	0	0	0
Maquinaria y equipos	0	0	0
Otros (definir)	509	0	509
TOTAL	**509**	**0**	**509**

COSTO PROYECTO I+D

FUENTE DE FINANCIAMIENTO	EN MONEDA NACIONAL	EN MONEDA EXTRANJERA	TOTAL
FONDEF	123	0	123
INSTITUCIONES	228	0	228
EMPRESAS Y OTRAS ENTIDADES	158	0	158
TOTAL	**509**	**0**	**509**

PARTE V ANEXOS

ANEXO 1.
PLAN DE NEGOCIOS

A continuación se describen los principales aspectos del Plan de Negocios en ejecución:

4.1 Productos

Se considera la comercialización de 3 servicios independiente. Cada uno de ellos se distribuye via Internet como módulos independientes de la plataforma RiskAmerica, la que al incluir estos nuevos módulos se distribuye bajo el nombre de RiskAmericaPlus.

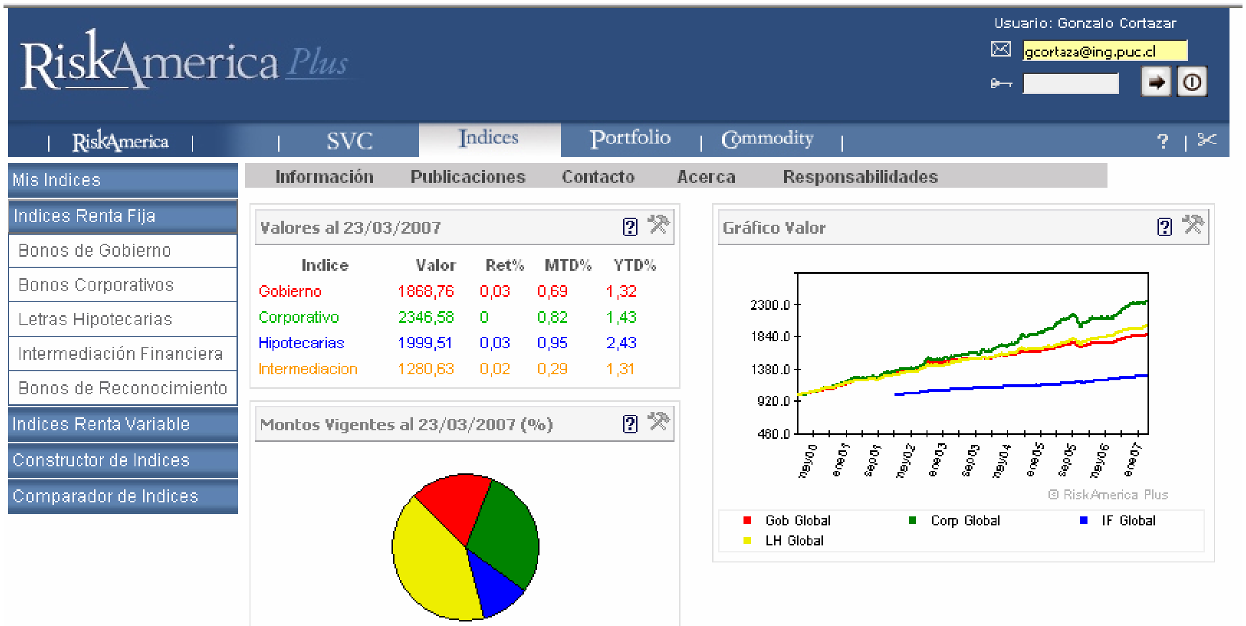
Los 3 Módulos de Servicios ofrecidos son:

4.1.1 Módulo SVC

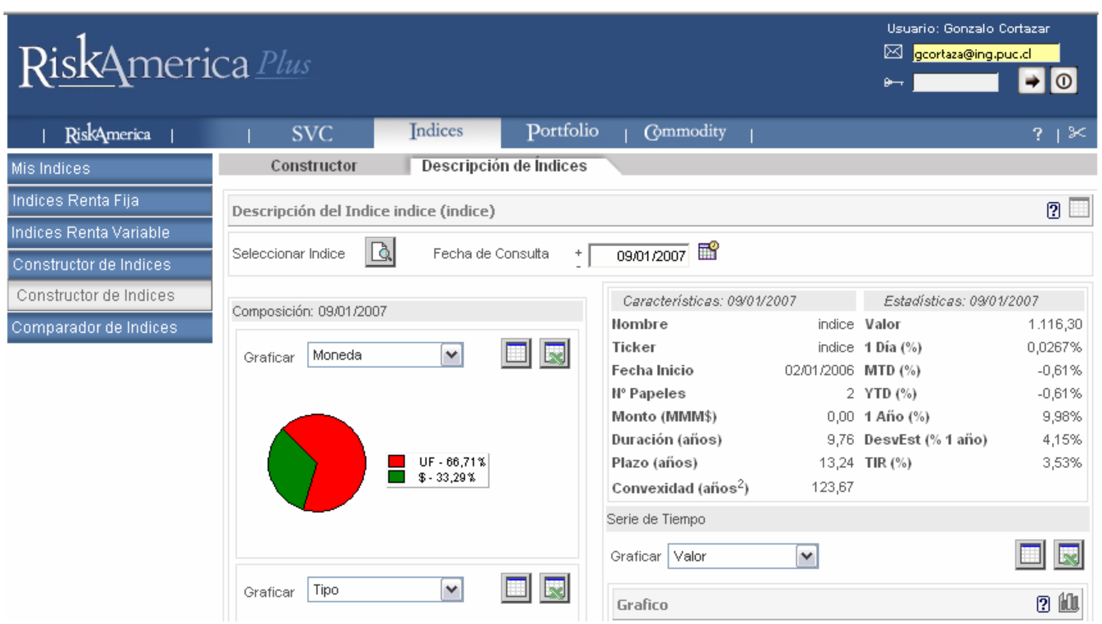
El SVC o Sistema de Valorización de Carteras consiste en un módulo del servicio RiskAmericaPlus cuyo objetivo es asignarle una TIR a cada nemotécnico solicitado.
El usuario envía vía Web un archivo indicando los nemotécnicos asociados a su cartera, devolviendo el sistema la TIR que el modelo le asigna a cada uno. La TIR del modelo depende de si el activo fue transado ese día, de cuál es la curva de referencia para el día (la que es actualizada diariamente) y de la historia de spreads que este nemotécnico ha exhibido respecto de la curva en el pasado.

4.1.2 Módulo Índices

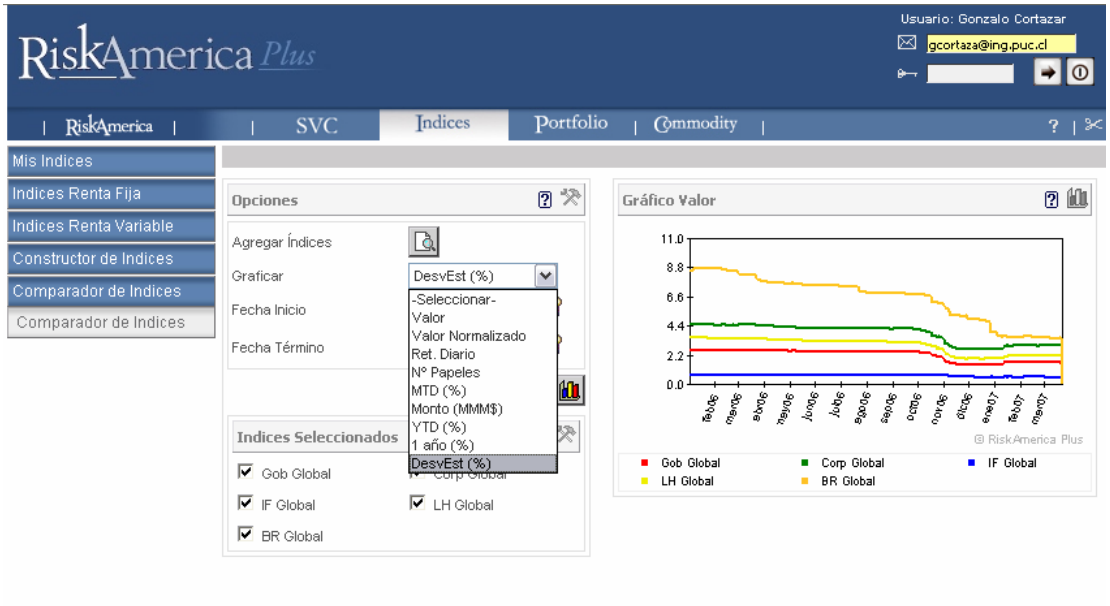
Este módulo entrega información referida al comportamiento del mercado financiero. Esta descripción se realiza en términos de distintas familias y clases de activos, incluyéndose tanto renta fija como variable.



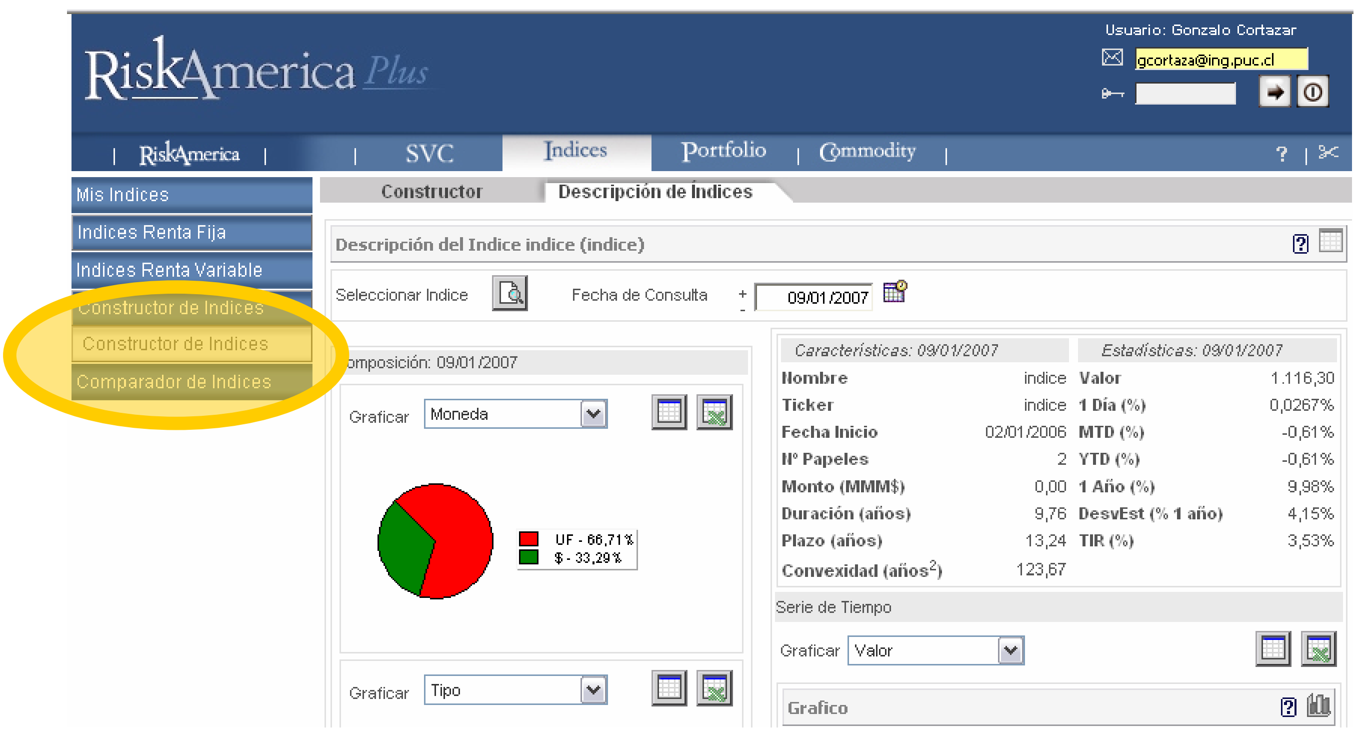
Para cada uno de los más de 100 índices existentes se entrega información de su composición así como de su comportamiento en términos de retornos y riesgos.



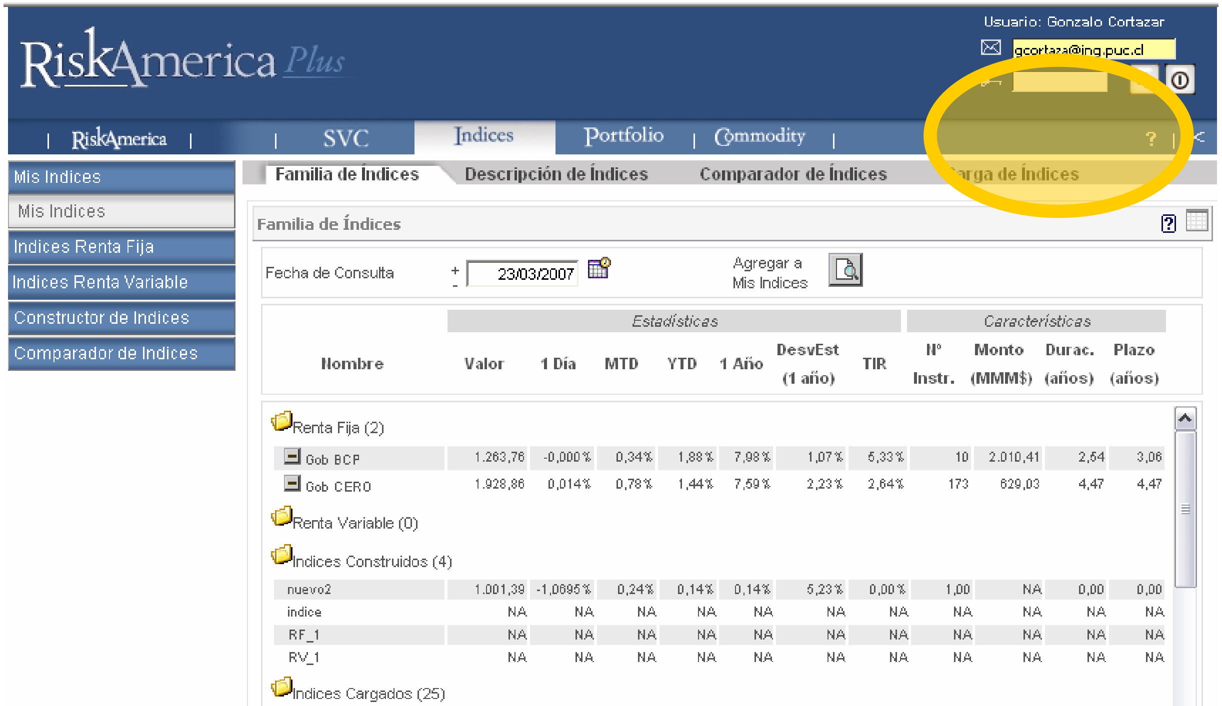
Asimismo, se pueden comparar y realizar análisis entre los distintos índices:



Se entrega además la posibilidad de construir y índices personalizados:



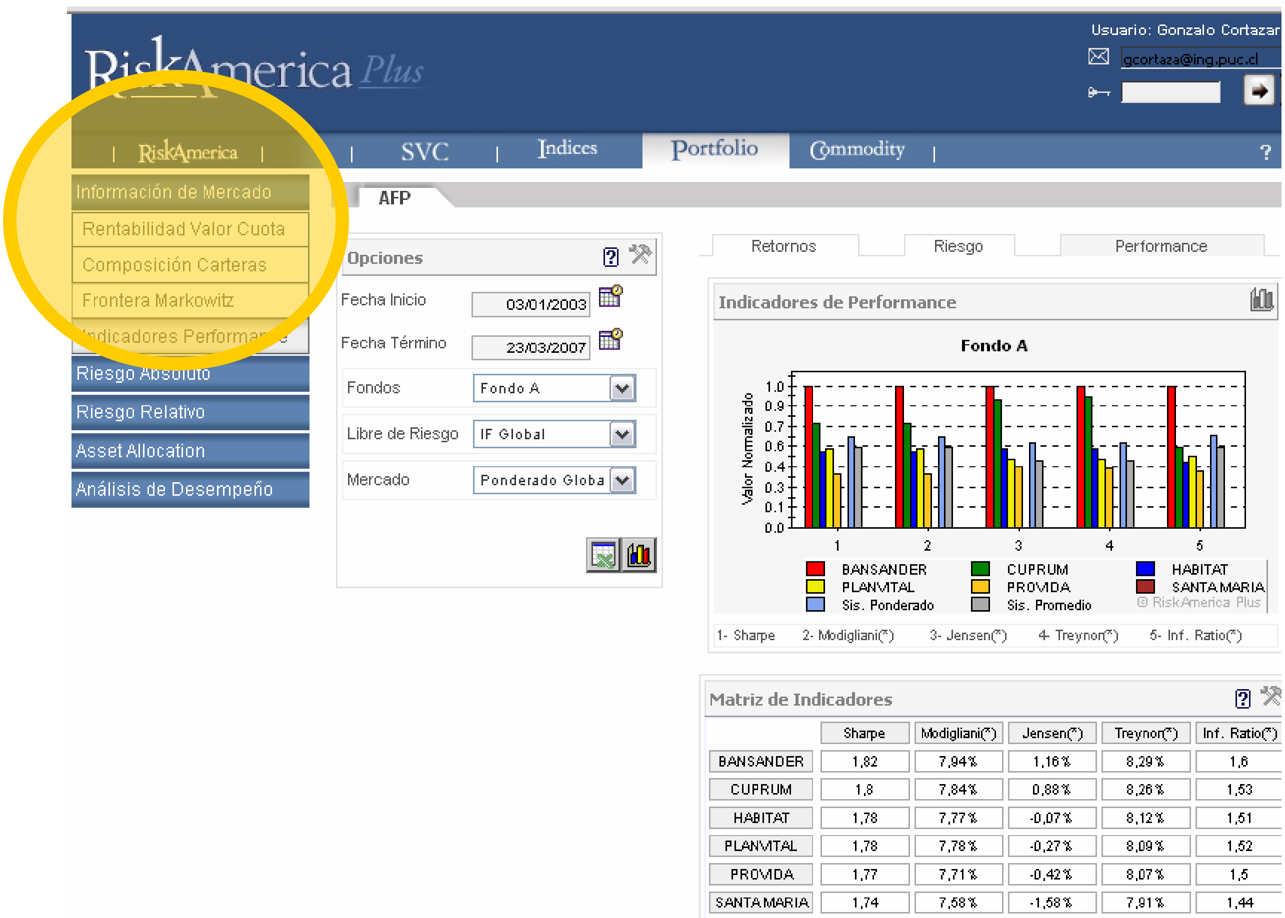
También se pueden cargar índices generados externamente:



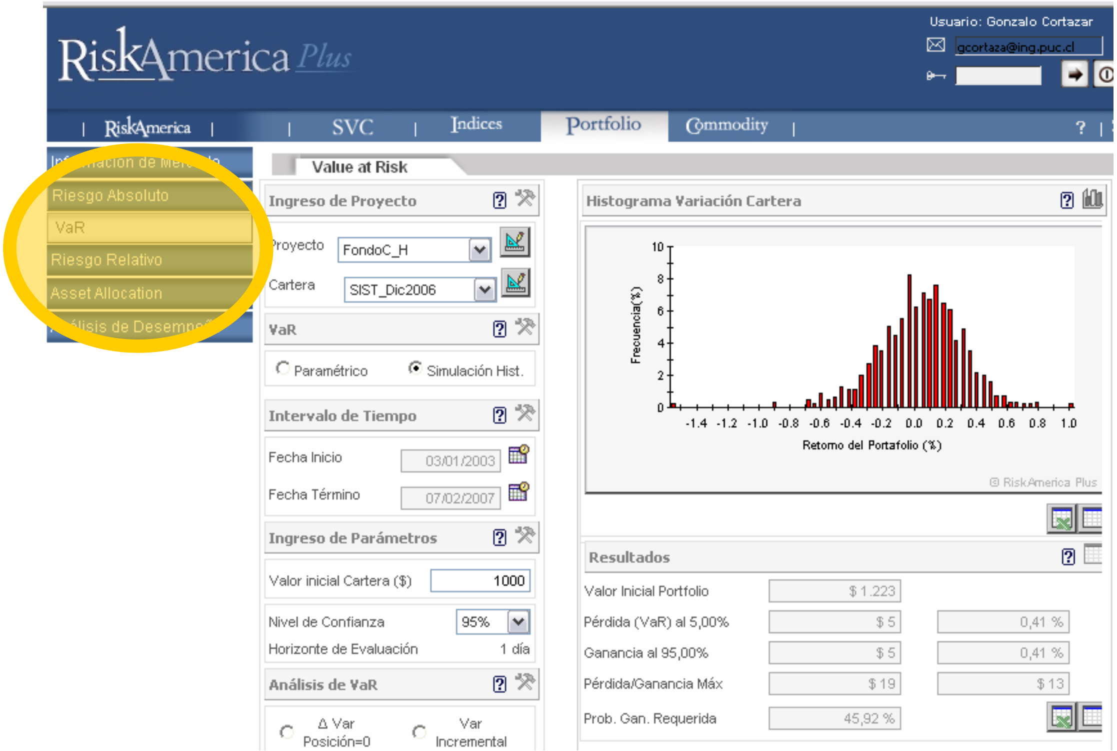
4.1.2 Módulo Portfolio

Este módulo entrega información de de riesgo, retorno y performance tanto de carteras existentes del mercado, como de carteras propias.

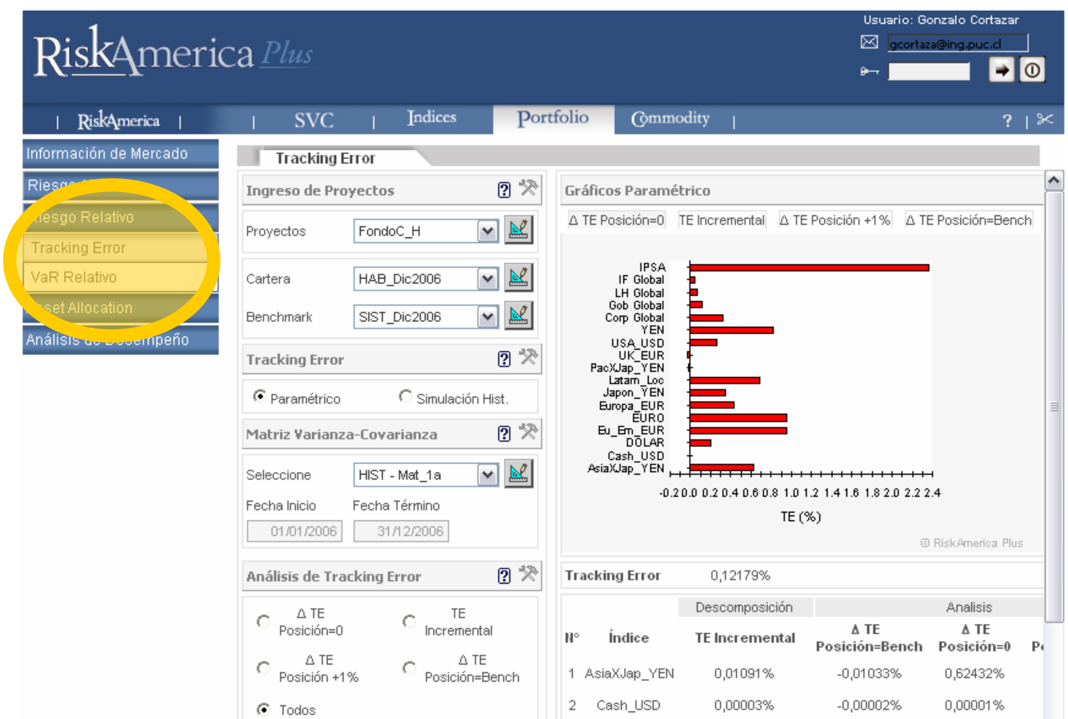
En *Información de Mercado* entrega información información de riesgo, de retornos y de performance de carteras públicas de AFP y de Fondos Mutuos, como se muestra en las siguientes páginas web:



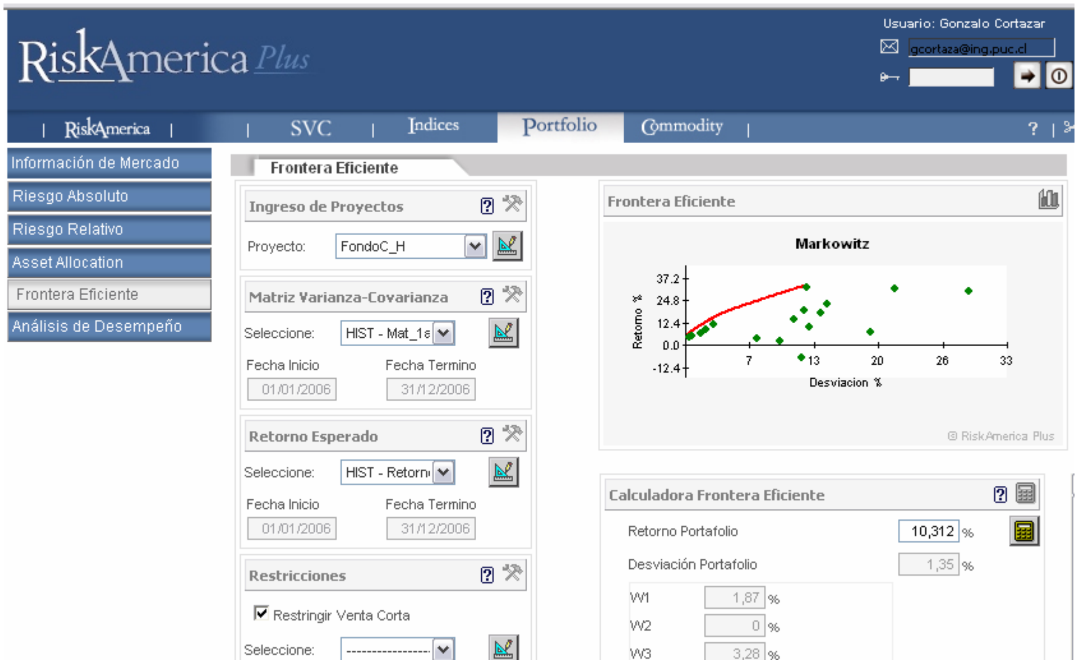
En *Riesgo Absoluto* se entregan herramientas para calcular el Value-at-Risk de carteras propias tanto por métodos paramétricos como por simulación histórica.



En *Riesgo Relativo* se entregan herramientas para calcular el Tracking Error y el VaR Relativo a entre dos carteras.



En *Asset Allocation* se entregan herramientas para calcular carteras con combinaciones riesgo-retorno óptimas.



En *Análisis de Desempeño* se entregan herramientas para calcular carteras el performance de carteras definidas por el usuario.

4.2 Clientes

Los clientes potenciales para los resultados de este proyecto son principalmente instituciones financieras a las que se pueden agregar sus instituciones reguladoras. A continuación se entrega una lista actualizada de los clientes potenciales provenientes de AFP, Fondos Mutuos, Bancos, Cias de Seguros. A estas listas habría que agregar corredoras de bolsa, securitizadoras, proveedores de diversos servicios financieros e instituciones reguladoras.

Tipo	Nombre
AFP	Bansander S.A.
	Cuprum S.A.
	Habitat S.A.
	Planvital S.A.
	Provida S.A.
	Santa María S.A.

Tipo	Nombre
Administradoras de Fondos Mutuos	Administradora General de Fondos Security S.A.
	Banchile Administradora General de Fondos S.A.
	BancoEstado S.A. Administradora General de Fondos
	Bandesarrollo Administradora General de Fondos S.A.
	BBVA Administradora General de Fondos S.A.
	BCI Administradora de Fondos Mutuos S.A.
	BICE Administradora General de Fondos S.A.
	Boston Administradora General de Fondos S.A.
	Celfín Capital S.A. Administradora General de Fondos
	Consorcio S.A. Administradora General de Fondos
	Corp Banca Administradora General de Fondos S.A.
	Cruz del Sur Administradora General de Fondos S.A.
	Euroamérica Administradora General de Fondos S.A.
	IM Trust S.A. Administradora General de Fondos
	Larraín Vial Administradora General de Fondos S.A.
	Legg Mason (Chile) Administradora General de Fondos S.A.
	Penta Administradora General de Fondos S.A.
	Principal Administradora General de Fondos S.A.
	Santander Santiago S.A. Administradora General de Fondos
	Scotia Sudamericano Administradora de Fondos Mutuos S.A.
	Zurich Administradora General de Fondos S.A.

Tipo	**Nombre**
Banco	ABN AMRO Bank
	Banco BICE
	Banco de Chile
	Banco de Crédito e Inversiones
	Banco de la Nación Argentina
	Banco del Desarrollo
	Banco del Estado de Chile
	Banco do Brasil
	Banco Falabella
	Banco Internacional
	Banco Paris
	Banco Penta
	Banco Ripley
	Banco Santander Santiago
	Banco Security
	BankBoston
	BBVA
	Citibank N.A.
	CorpBanca
	Deutsche Bank
	HNS Banco
	HSBC Bank Chile
	JPMorgan Chase Bank
	Scotiabank Sud Americano
	The Bank of Tokyo-Mitsubishi Ltd.

Tipo	**Nombre**
Seguros	Aseguradora Magallanes
	BICE Vida
	Cardif
	Chilena Consolidada
	COFACE
	Consorcio Nacional
	Continental
	Corredora Security S.A.
	Cruz del Sur
	Eurovida
	ING
	Interamericana
	ISE
	Liberty
	MAPFRE
	Mutual de Carabineros
	Mutual de Seguros de Chile
	Renta Nacional
	Royal & Sunalliance
	Seguros de Vida la Construcción SA

4.3 Factores de Éxito

El principal Factor de Éxito es la capacidad de generar y comunicar una reputación de objetividad, rigurosidad y compromiso de permanente innovación para los Servicios ofrecidos. Para ello se hace necesario mantener activo un equipo de investigadores haciendo investigación reconocida internacionalmente.

Es en este contexto que independientemente de la evolución comercial de RiskAmerica, ésta se debe mantener ligada estrechamente a la investigación académica que se realiza en la Pontificia Universidad Católica de Chile a través del FINlabUC-Laboratorio de Investigación Avanzada en Finanzas.

4.4 Comercialización

La distribución de los resultados se realizará a través de la plataforma WEB RiskAmerica, la que comunicó al mercado la incorporación de estos servicios expandidos como RiskAmericaPlus.
Asimismo se deben intensificar el plan de contactos, comunicación y capacitación de usuarios potenciales a través de encuentros y presentaciones y distribución de material gráfico.
Adicionalmente se debe seguir explorando oportunidades de internacionalización de los servicios.

A continuación se entrega copia parcial de material desarrollado.

4.5 Ventas Actuales y Proyección de Resultados Futuros

Las ventas esperadas de los 3 módulos de Servicios para el año 2007 alcanzan MM$198 las que debieran incrementarse en los años futuros hasta alcanzar un monto esperado de MM$570 el año 2010.

A continuación se presentan los ingresos y egresos proyectados por los próximos 7 años, así como el VAN de los excedentes descontados al 12% anual.

En forma muy conservadora, no se supone valor residual alguno.

Se estima un VAN al 12% de **MM$781,6 para** la explotación comercial de los resultados del proyecto

Montos en MM$

	2007	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**
Mod-SVC	180	200,0	220,0	240	240	240	240
Mod-Indices	10	47,0	85,0	120	120	120	120
Mod-Portfolio	8	75,0	142,0	210	210	210	210
Total Ing	**198**	**322**	**447**	**570**	**570**	**570**	**570**
Costos Fijos	120	120	120	120	120	120	120
Costos Variables	65,34	106,26	147,51	188,1	188,1	188,1	188,1
Total Costos	**185,34**	**226,26**	**267,51**	**308,1**	**308,1**	**308,1**	**308,1**
Excedentes	**12,66**	**95,74**	**179,49**	**261,9**	**261,9**	**261,9**	**261,9**
VAN(12%)	**781,6**						

ANEXO 2. PLANES DE TRABAJO INICIAL Y EFECTIVAMENTE EJECUTADO

Tipo	Nombre Resultado	Fechas			Descripción
		Programada	Reprogramada	Lograda	
R2	**Convenios Contraparte** : Convenios Con Contrapartes	06/03/2005	06/04/2005	06/04/2005	
H19	**Plan de Experimentos definido** : Plan De Experimentos Definido A Www.riskportfolio	30/06/2005	30/06/2005	30/06/2005	
H19	**Diseño de Prototipo emitido** : Diseno De Pagina Web	30/06/2005	30/06/2005	30/06/2005	
H19	**Experimentos Críticos Efectuados** : Experimentos Criticos Efectuados Www.riskportfolio	30/06/2005	15/09/2005	15/09/2005	
H19	**Prototipo Probado a nivel Laboratorio** : Prototipo Www.riskportfolio Probado A Nivel Laboratorio	30/06/2005	30/09/2005	30/09/2005	
R19	**Prototipo probado a Nivel Piloto/Planta** : Www.riskportfolio.comv1	30/06/2005	30/09/2005	30/09/2005	
H27	**Diseño de Prototipo emitido** : Diseno De Assetallocationv1	01/08/2005	01/08/2005	01/08/2005	
H27	**Plan de Experimentos definido** : Plan De Experimentos Definido Para Assetallocation	15/08/2005	15/08/2005	15/08/2005	
H27	**Experimentos Críticos Efectuados** : Experimentos Criticos Efectuados Assetallocation	25/08/2005	25/08/2005	25/08/2005	
H27	**Prototipo Probado a nivel Laboratorio** : Prototipo Probado A Nivel Laboratorio Assetallocation	30/09/2005	30/09/2005	30/09/2005	
R27	**Prototipo probado a Nivel Piloto/Planta** : Assetallocationv1	30/09/2005	30/09/2005	30/09/2005	
H29	**Plan de Experimentos definido** : Plan De Experimentos Definido Riskmatrix	15/10/2005	15/10/2005	15/10/2005	

H 21	**Plan de Experimentos definido** : Plan De Experimentos Definido Portfoliovalue	15/10/2005	15/10/2005	15/10/2005	
H 21	**Experimentos Críticos Efectuados** : Experimentos Criticos Efectuados Portfoliovalue	25/10/2005	25/10/2005	25/10/2005	
H 29	**Experimentos Críticos Efectuados** : Experimentos Criticos Efectuados Riskmatrix	25/10/2005	25/10/2005	25/10/2005	
H 29	**Diseño de Prototipo emitido** : Diseno De Riskmatrixv1	30/10/2005	30/10/2005	30/10/2005	
H 21	**Diseño de Prototipo emitido** : Diseno De Portfoliovaluev1	30/10/2005	30/10/2005	30/10/2005	
H 21	**Prototipo Probado a nivel Laboratorio** : Prototipo Probado A Nivel Laboratorio Portfoliovalue	30/12/2005	15/03/2006	23/01/2006	
R 21	**Prototipo probado a Nivel Piloto/Planta** : Portfoliovaluev1	30/12/2005	15/03/2006	23/01/2006	
R 29	**Prototipo probado a Nivel Piloto/Planta** : Riskmatrixv1	30/12/2005	30/12/2006	30/12/2006	
H 29	**Prototipo Probado a nivel Laboratorio** : Prototipo Probado A Nivel Laboratorio Riskmatrix	30/12/2005	30/12/2006	30/12/2006	
H 25	**Plan de Experimentos definido** : Plan De Experimentos Definido Portfoliobenchmarks	15/04/2006	15/04/2006	28/04/2006	
H 23	**Plan de Experimentos definido** : Plan De Experimentos Definido Portfoliorisk	15/04/2006	15/06/2006	15/06/2006	
H 25	**Experimentos Críticos Efectuados** : Experimentos Criticos Efectuados Portfoliobenchmarks	25/04/2006	25/04/2006	28/04/2006	
H 23	**Experimentos Críticos Efectuados** : Experimentos Criticos Efectuados Portfoliorisk	25/04/2006	30/06/2006	30/06/2006	
H 25	**Diseño de Prototipo emitido** :	30/04/2006	30/04/2006	28/04/2006	

	Diseno De Portfoliobenchmarksv1				
23	**Diseño de Prototipo emitido** : Diseno De Portfolioriskv1	30/04/2006	30/07/2006	30/06/2006	
16	**Tesís Iniciada** : Inicio Tesis Pregrado	30/06/2006	30/06/2006	30/06/2006	
17	**Publicación solicitada** : Documento De Trabajo De Publicacion	30/06/2006	30/06/2006	30/06/2006	
25	**Prototipo Probado a nivel Laboratorio** : Prototipo Probado A Nivel Laboratorio Portfoliobenchmarks	30/06/2006	30/06/2006	30/06/2006	
23	**Prototipo Probado a nivel Laboratorio** : Prototipo Probado A Nivel Laboratorio Portfoliorisk	30/06/2006	30/06/2006	30/06/2006	
15	**Tesís Iniciada** : Inicio Tesis De Postgrado	30/06/2006	30/06/2006	30/06/2006	
23	**Prototipo probado a Nivel Piloto/Planta** : Portfolioriskv1	30/06/2006	30/08/2006	01/08/2006	
18	**Evento Programado** : Programacion Evento De Difusion	30/06/2006	30/12/2006	10/01/2007	
25	**Prototipo probado a Nivel Piloto/Planta** : Portfoliobenchmarksv1	30/06/2006	02/01/2007	02/01/2007	
31	**Plan de Experimentos definido** : Plan De Experimentos Definido Productofinal	15/12/2006	30/12/2006	30/12/2006	
31	**Experimentos Críticos Efectuados** : Experimentos Criticos Efectuados Productofinal	25/12/2006	30/12/2006	20/01/2007	
16	**Tesís** : 2 Tesis De Pregrado	30/12/2006	30/12/2006	31/12/2005	
15	**Tesís** : Tesis De Magister	30/12/2006	30/12/2006	17/01/2006	
31	**Diseño de Prototipo emitido** : Diseno De Productofinalriskportfoliov2	30/12/2006	30/12/2006	20/01/2007	
18	**Eventos** : Evento De Difusion	30/12/2006	30/12/2006	29/03/2007	
17	**Publicaciones** : Publicaciones Resultados	30/12/2006	30/12/2006	17/04/2007	

31	**Prototipo Probado a nivel Laboratorio** : Prototipo Probado A Nivel Laboratorio Productofinal	30/01/2007	30/01/2007	25/04/2007	
31	**Prototipo Probado a nivel Piloto/Planta** : Prototipo Probado A Nivel Piloto/planta Productofinal	05/03/2007	05/03/2007	30/03/2007	

Reprogramados: **(Este proyecto no ha reprogramado resultados)**

Eliminados: **(Ningún resultado ha sido eliminado)**

No Logrados: **(No hay resultados No Logrados)**

Detalle de Resultados: **(No hay resultados ingresados para este proyecto)**

ANEXO 3. PLANILLA PRESUPUESTARIA INICIAL Y DE EJECUCIÓN TOTAL DEL PROYECTO *(incorpore planillas presupuestarias inicial y final detalladas por Ítem y fuente de financiamiento)*

PLANILLA PRESUPUESTARIA INICIAL

		FINANCIAMIENTO		
ITEM	COSTO TOTAL	INSTITUCIONES	EMPRESAS U OTRAS ENTIDADES	FONDEF
HONORARIOS, INCENTIVOS, REMUNERACIONES	278,738	67,789	87,525	123,424
SUBCONTRATOS	0,000	0,000	0,000	0,000
CAPACITACION	0,000	0,000	0,000	0,000
PASAJES Y VIATICOS	13,080	0,000	9,319	3,761
EQUIPOS	35,161	25,361	5,000	4,800
INFRAESTRUCTURA	29,700	29,700	0,000	0,000
SOFTWARE	0,000	0,000	0,000	0,000
FUNGIBLES	2,700	0,000	1,350	1,350
PUBLICACIONES Y SEMINARIOS	0,000	0,000	0,000	0,000
PROPIEDAD INTELECTUAL	126,500	0,000	125,000	1,500
GASTOS COMUNES	2,697	0,000	0,000	2,697
GASTOS GENERALES	9,070	0,000	0,306	8,764
GASTOS DE ADMINISTRACION SUPERIOR	11,704	0,000	0,000	11,704
TOTAL	**509,350**	**122,850**	**228,500**	**158,000**

PLANILLA PRESUPUESTARIA FINAL

		FINANCIAMIENTO		
ITEM	COSTO TOTAL	INSTITUCIONES	EMPRESAS U OTRAS ENTIDADES	FONDEF
HONORARIOS, INCENTIVOS, REMUNERACIONES	280,122	67,759	78,521	133,842
SUBCONTRATOS				
CAPACITACION				
PASAJES Y VIATICOS	7,697		3,936	3,761
EQUIPOS	35,781	25,414	6,204	4,163
INFRAESTRUCTURA	41,519	29,700	11,819	
SOFTWARE	0,169		0,169	
FUNGIBLES	1,050		0,953	0,097
PUBLICACIONES Y SEMINARIOS				
PROPIEDAD INTELECTUAL	125,001		125,001	
GASTOS COMUNES				
GASTOS GENERALES	3,374		3,309	0,065
GASTOS DE ADMINISTRACION SUPERIOR	11,704			11,704
TOTAL	**506,417**	**122,873**	**229,912**	**153,632**

ANEXO 4. INFRAESTRUCTURA Y BIENES ADQUIRIDOS POR EL PROYECTO

Para cada obra de infraestructura o equipo cuyo valor facturado ***sea mayor a US$ 5.000*** *identifique los usos, responsables y porcentaje de tiempo en que será utilizado, de modo que su uso sea coherente con la línea de trabajo del proyecto. El plan de mantención se debe realizar según estándares.*

1.- Listado de obras de infraestructura

(listado definitivo identificando el nombre de la obra, características, superficie construida, la unidad institucional que la utiliza y la dirección del lugar en que se encuentra)

Nombre de la infraestructura	**Características de la construcción**	**Superficie**	**Unidad Institucional responsable**	**Dirección** *(calle, Nº, ciudad)*

2.- Listados de bienes (equipos y otros)

(listado definitivo de equipos identificando el nombre, características y código del equipo y de inventario, precio facturado, la unidad institucional a que está asignado, el responsable de la unidad y la dirección del lugar en que se encuentra)

Nº	**Nombre del equipo**	**Marca**	**Serie**	**Modelo**	**Nº inventario**	**Precio de compra MM$**
1	Proyector Multimedia	Nec		VT 670	73008	776
1	Computador	Armado		Intel Pentium IV	74884	1,487
4	Computadores	Armados		Intel Pentium IV	73736 – 73737 – 73738 – 73739	1,900

(continuación tabla anterior)

Nº	**Responsable** *(nombre completo)*	**Unidad Institucional**	**Dirección** *(calle Nª, ciudad)*	**Usos***	**% Estimado de uso**
1	Gonzalo Cortazar Sanz	P.U.C.	Avda. Vicuña Mackenna Nº 4860		

* USOS: (1) Docencia , (2) Investigación, (3) Servicios, (4) Capacitación,(5) Asesorías , Otros describir

3.- PLAN DE MANTENCIÓN. *El contrato de finiquito incluirá el plan de mantenimiento, operación y cuidado de equipos y mantención de obras así como los seguros de rigor.*

Nombre del equipo	**Nº inventario**	**Actividades principales de mantención**	**Período entre mantenciones**	**Responsable**

ANEXO 5. PUBLICACIONES

Cortazar, G., Gravet, M., Urzua, J. (2008) “The Valuation of Multidimensional American Real Options using the LSM Simulation Method” *Computers & Operations Research* Vol 35 (2008) 113 – 129

Cortazar, G, Schwartz, E. S., Naranjo, L, (2007) "Term Structure Estimation in Markets with Infrequent Trading“ *International Journal of Finance and Economics* (por aparecer)

Cortazar, G., Naranjo, L. (2006) “An N-Factor Gaussian Model of Oil Futures Prices” *The Journal of Futures Markets*, Vol.26, No. 3, March, 2006, 243-268

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TERM-STRUCTURE ESTIMATION IN MARKETS WITH INFREQUENT TRADING

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ABSTRACT

There are two issues that are of central importance in term-structure analysis. One is the modelling and estimation of the current term structure of spot rates. The second is the modelling and estimation of the dynamics of the term structure. These two issues have been addressed independently in the literature. The methods that have been proposed assume a sufficiently complete price data set and are generally implemented separately. However, there are serious problems when these methods are applied to markets with sparse bond prices.

We develop a method for jointly estimating the current term-structure and its dynamics for markets with infrequent trading. We propose solving both issues by using a dynamic term-structure model estimated from incomplete panel-data. To achieve this, we modify the standard Kalman filter approach to deal with the missing-observation problem. In this way, we can use historic price data in a dynamic model to estimate the current term structure. With this approach we are able to obtain an estimate of the current term structure even for days with an arbitrary low number of price observations.

The proposed methodology can be applied to a broad class of continuous-time term-structure models with any number of stochastic factors. To show the implementation of the approach, we estimate a three-factor generalized-Vasicek model using Chilean government bond price data. The approach, however, may be used in any market with infrequent trading, a common characteristic of many emerging markets.Copyright © 2007 John Wiley & Sons, Ltd.

JEL CODE: ■; ■; ■

KEY WORDS: ■; ■; ■

1. INTRODUCTION

There are two issues that are of central importance in term-structure analysis. One is the modelling and estimation of the current term structure of spot rates, which is essential for valuing and hedging cash flows that are linearly related to the discount function. The second is the modelling and estimation of the dynamics of the term structure, which is required for valuing and hedging cash flows that are non-linear functions of the term structure (all types of options). These two issues have been addressed independently in the literature.

For current term-structure estimation, most authors have proposed parametric and non-parametric methods for fitting curves to current bond prices (or yields) without regard to past prices. McCuloch (1971, 1975), Vasicek and Fong (1982) and Fisher *et al*. (1994), among others, use spline curve-fitting methods to estimate the current term structure. Nelson and Siegel (1987) and Svensson (1994) use parsimonious

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representations of the yield curve, limiting the number of parameters and giving more stability to the term structure.

For the modelling of the term-structure dynamics the main concern is the movement of the term structure across time. To address this issue one alternative is to model the stochastic movement of the spot rate and then to use no-arbitrage arguments to infer the dynamics of the term structure. Examples of this approach include one-factor mean-reverting models (Vasicek, 1977), two-factor models (Brennan and Schwartz, 1979), multifactor extensions of the Vasicek model (Langetieg, 1980), single-factor general equilibrium models (Cox *et al*., 1985) and multi-factor extensions of the CIR model (Duffie and Kan, 1996), among many others. Another approach is to use the whole current term structure as the input to the model and no-arbitrage arguments to infer its stochastic movement (Ho and Lee, 1986; Heath *et al*., 1992). Even though these type of models use all the information contained in the current term structure they are more difficult to calibrate.

Once a dynamic model of interest rates is proposed, the estimation method that will be used must be chosen. One possibility is to estimate the model using a time-series of bond prices (Chan *et al*., 1992; Broze *et al*., 1995; Brenner *et al*., 1996; Nowman, 1997, 1998; Andersen and Lund, 1997). Alternatively, state variables and parameters may be estimated from a panel of bond prices with different maturities (Chen and Scott, 1993; Pearson and Sun, 1994; Duffie and Singleton, 1997).

Even though there are obvious benefits of calibrating a model using a panel with a large number of price observations, the richer the data set, the larger the estimated measurement errors. These errors arise from the inability of a model with a limited number of factors to perfectly explain a large number of contemporaneous prices. A powerful and widely used methodology to optimally estimate unobservable state variables from a noisy panel-data is the Kalman filter. Recent applications of this methodology to dynamic models of interest rates include Lund (1994, 1997), Ball and Torous (1996), Duan and Simonato (1999), Geyer and Pichler (1999), Babbs and Nowman (1999) and Chen and Scott (2003). The advantage of using the Kalman filter on a panel-data is that it jointly uses all present and past price information. Maximum likelihood methods can then be used to estimate the parameters of the model.

Both type of methods proposed in the literature, curve fitting for estimating the current term structure and Kalman filtering for dynamic models, have been successfully applied to markets for which there is a sufficiently complete price data set. However, there are serious problems when these methods are used in markets with sparse bond price data. For example, traditional curve-fitting methods render unreliable estimates of the current term structure for days without a sufficient number of observations or without short or long-term bond prices. In addition, a typical Kalman filter implementation assumes a complete panel of bond prices (or yields), which becomes problematic if there is a substantial number of missing observations as is the case in many emerging markets.

In this article we develop and implement a method for jointly estimating the current term structure and its dynamics in markets with infrequent trading. We propose solving both issues by using a dynamic term-structure model estimated from incomplete panel-data. To achieve this, we modify the standard Kalman filter approach to deal with the missing-observation problem. We can then use historical price data and a dynamic model to estimate the current term structure. With this approach, we are able to obtain an estimate of the current term structure even for days with an arbitrary low number of price observations.

The proposed methodology can be applied to a broad class of continuous-time term-structure models with any number of stochastic factors. To show the implementation of the approach for an emerging market with infrequent trading, we estimate a three-factor generalized-Vasicek model using Chilean government bond price data. The approach, however, may be used in any market with infrequent trading as is the case in many emerging markets.

The next section explains the shortcomings of static term-structure estimation methods when there is sparse data. In Section 3 we present the generalized-Vasicek model that will be used for illustrating our methodology. Section 4 presents the standard Kalman filter method and shows how it can be used in an incomplete panel-data setting. Section 5 presents empirical results of applying the methodology to the Chilean government bond market and Section 6 concludes.

2. SHORTCOMINGS OF STATIC TERM-STRUCTURE ESTIMATION MARKETS WITH INFREQUENT TRADING

Term-structure estimation has been traditionally implemented with static models that only use current bond prices (or yields) without regard to past information. Some methods, like Nelson and Siegel (1987) and Svensson (1994), assume a parametric functional form for the forward rates.[1] Other methods, for example, McCuloch (1971, 1975), and Fisher *et al.* (1994), use non-parametric spline-based interpolation methods to calculate the term structure. Empirical evidence shows that in well-developed markets, where numerous bonds are traded every day for different maturities, these static methods generate yield curves that accurately fit current bond transactions (Bliss, 1996).

There are, however, other features besides goodness of fit to observed prices that are desirable in a term-structure model, such as the time-series stability of the term-structure curves obtained. This stability can be analysed by observing the sequence of daily term-structure estimations implied by the model. It might well be the case that the model fits very well the existing bond prices (or yields), but it implies large daily movements of yields for maturities that are not traded. This is not an issue for liquid markets, but as we shall see, is a serious problem for thin markets. One way of assessing the stability of the term-structure curves obtained is to compare the volatilities from the model with actual volatility from the data.

In markets with a complete cross-section of prices for each date, volatility of interest rates computed from the estimated term structures will closely match historical data and the stability of the model is not an issue. However, for sparse data sets in which at each date there are only a few different bond maturities traded, stability will become an important criterion for judging the reliability of the term-structure estimation.

When the number of observed prices for a particular date is not sufficiently larger than the number of parameters to be estimated, any measurement error crucially affects the shape of the fitted curve. An extreme case is when the number of parameters to be estimated is larger than the number of observed prices; in this case there is an infinite number of curves that fit the observed prices. Figure 1 illustrates this extreme (but not uncommon in emerging markets) case of a date with fewer prices than model parameters by plotting two of the infinite term-structures that perfectly explain observed prices. This example is taken from one of the many dates in the Chilean government bond market with extremely thin trading. Curve-fitting methods clearly cannot be applied to dates with very low number of transactions.

A second problem of these static curve-fitting methods when used in markets with infrequent trading occurs when the prices for short or for long-term bonds are not available, even if the number of observed prices is sufficient for the estimation. Curve-fitting methods provide reasonable estimates within the time range spanned by the available prices, but provide much less reliable estimates for extrapolations outside this range. In many emerging markets it is common that for some dates long-term bonds are not traded; but the need for a complete term-structure estimation for valuation and hedging purposes remains.

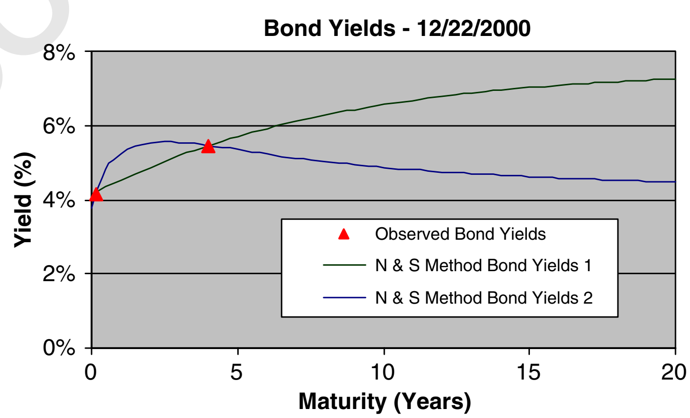

Figure 1. Two different estimations of yield curves from Chilean government inflation-protected discount and coupon bond data using the Nelson and Siegel method for 12/22/2000.

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Figure 2 illustrates a 20-year term-structure estimate of the coupon-bond-yield in Chile for 10/06/1999, a date in which there are sufficient bond prices but the maturity of the longest bond traded was only 6 years. We use all pure-discount and coupon bonds[2] traded on that date to compute the implied pure-discount yield curve using the Svensson (1994) method. Once this curve is obtained we compute the yields of coupon bonds with maturities from 0.5 to 20 years priced using the implied pure-discount yield curve estimated earlier. This coupon-yield curve is then plotted in Figure 2 together with the yields of all market transactions on 10/06/1999 and on the day before.

From Figure 2 we can see that prices of traded bonds with similar maturities did not change much between both dates and that long-term bonds were traded only on the first day. Even though observed prices indicate that markets seem to have behaved similarly on both dates, the model estimates that the yield of a 19-year coupon bond changed by almost 1% in a day. The extrapolated 19-year yield is clearly inaccurate. Curve-fitting methods provide unstable estimates of long rates when no long-term bonds are traded.

Instability of term-structure estimates can be measured by comparing the volatility term-structure implied by the model with the empirical volatilities obtained from the time-series of yields. It is well-known that the term structure of volatilities is downward sloping due to mean reversion in interest rates. This means that the volatility of long rates obtained from the model should be lower than the volatility of short rates.

Figure 3 plots the volatility of interest rates calculated from daily estimations of the term-structure in Chile between 1997 and 2001 using the Svensson (1994) method. It can be seen that this term-structure of

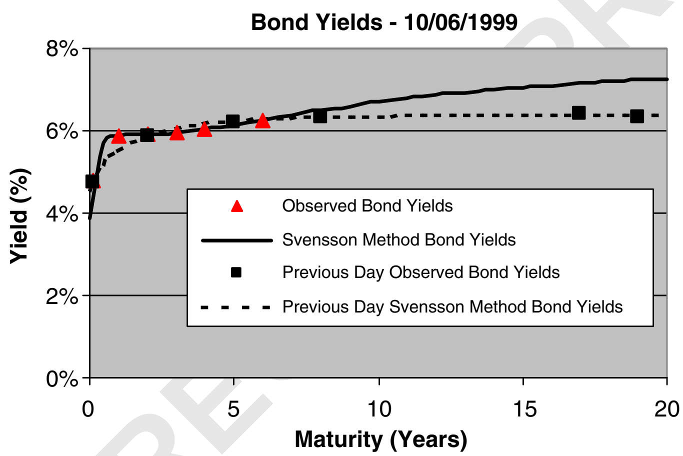

Figure 2. Coupon-bond-yields for two consecutive dates (10/05/1999 and 10/06/1999) estimated from Chilean government inflation-protected discount and coupon bond data using the Svensson (1994) method.

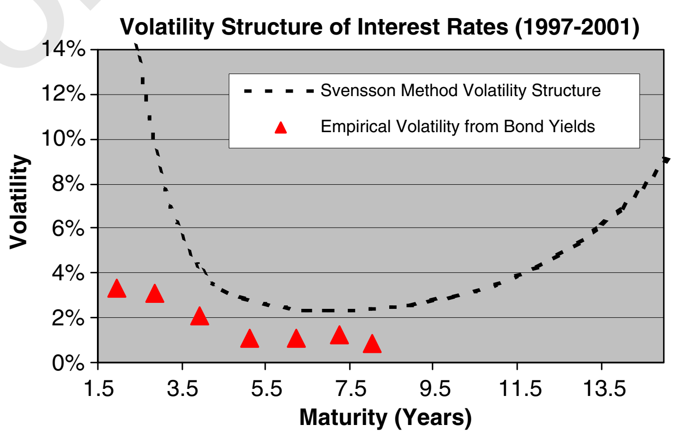

Figure 3. Empirical volatilities of interest rates in Chile and volatilities obtained from daily estimations of the term-structure between 1997 and 2001 using the Svensson (1994) method.

volatilities is not consistent with mean reversion in interest rates: it implies very high volatilities for long rates. Moreover, the Svensson volatility estimates are much higher than the empirical estimates obtained directly from bond prices, suggesting that missing observations induce unreliable rate estimates. Similar results are obtained when using other curve-fitting methods like Nelson and Siegel (1987).

3. THE GENERALIZED-VASICEK DYNAMIC TERM-STRUCTURE MODEL

As was shown in the previous section, traditional static term-structure estimation only incorporates current bond price (or yield) observations, without regard to past information. When long-term bond prices are not available, the estimation of long-term interest rates becomes unreliable. Also, without a sufficient number of transactions an over-parameterization of traditional models can occur.

We propose to solve the problems of term-structure estimation in markets with infrequent trading by using also past price information to infer the current term structure. This requires a dynamic model of the stochastic behaviour of interest rates to be able to mix current and past prices in a meaningful way.

Some dynamic models, in particular multifactor ones, use a limited number of unobservable factors to summarize the stochastic behaviour of the whole yield curve in a way that is sufficiently accurate, but also tractable. These unobservable state variables, together with the model parameters, must be estimated using observable bond price information. In the following sections we present an estimation methodology, based in the Kalman filter, that may be successfully used to estimate the term structure in markets with infrequent trading. To illustrate our estimation methodology we will consider a generalized Vacisek model for the instantaneous risk-free interest rate. Our methodology may be used, however, with other interest rate models such as a one-factor CIR model (Cox *et al.*, 1985), a multifactor CIR model (Duffie and Kan, 1996) or general exponential-affine models (Dai and Singleton, 2000), among others.

A generalized-Vasicek model is a multifactor mean-reverting Gaussian model of the instantaneous spot interest rate which extends Vasicek (1977). This generalized formulation goes back to Langetieg (1980), and is also analysed in Babbs and Nowman (1999). It considers n stochastic mean-reverting factors represented by the vector $\mathbf{x}_t$, of dimension $n \times 1$, that define the instantaneous interest rate r_t

$$r_t = \mathbf{1}'\mathbf{x}_t + \delta \tag{1}$$

The vector of state variables $\mathbf{x}_t$ is governed by the following stochastic differential equation:

$$\mathrm{d}\mathbf{x}_t = -\mathbf{K}\mathbf{x}_t\,\mathrm{d}t + \mathbf{\Sigma}\mathrm{d}\mathbf{w}_t \tag{2}$$

where $\mathbf{K} = \mathrm{diag}(k_i)$ and $\mathbf{\Sigma} = \mathrm{diag}(\sigma_i)$ are $n \times n$ diagonal matrices with entries that are strictly positive constants and different. Also, $\mathrm{d}\mathbf{w}_t$ is a $n \times 1$ vector of correlated Brownian motion increments such that

$$(\mathrm{d}\mathbf{w}_t)'(\mathrm{d}\mathbf{w}_t) = \mathbf{\Omega}\,\mathrm{d}t \tag{3}$$

where the (i,j) element of $\mathbf{\Omega}$ is $\rho_{ij} \in [1,-1]$, the instantaneous correlation of state variables i and j. Under this specification, the state variables have the multivariate normal distribution and each of them reverts to 0, at a mean reversion rate[3] given by k_i. Thus, according to equation (1) the instantaneous interest rate reverts to a long-term value given by the constant δ. Note that this is a canonical model in the sense that it contains the minimum number of parameters that can be econometrically identified (see Dai and Singleton, 2000).[4]

By assuming constant risk premiums[5] $\boldsymbol{\lambda}$, the risk-adjusted process for the vector of the state variables is

$$\mathrm{d}\mathbf{x}_t = -(\boldsymbol{\lambda} + \mathbf{K}\mathbf{x}_t)\,\mathrm{d}t + \mathbf{\Sigma}\mathrm{d}\mathbf{w}_t \tag{4}$$

where $\boldsymbol{\lambda}$ is a $n \times 1$ vector of constants.

Applying standard no-arbitrage arguments, we obtain the value of a pure-discount bond $P(\mathbf{x}_t, t)$

$$P(\mathbf{x}_t, \tau) = \exp(\mathbf{u}(\tau)'\mathbf{x}_t + v(\tau)) \tag{5}$$

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where

$$u_i(\tau) = -\frac{1-\exp(-k_i\tau)}{k_i} \tag{6}$$

$$\begin{aligned} v(\tau) = {} & \sum_{i=1}^{N}\frac{\lambda_i}{k_i}\left(\tau - \frac{1-\exp(-k_i\tau)}{k_i}\right) - \delta\cdot\tau \\ & + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\sigma_i\sigma_j\rho_{ij}}{k_ik_j}\left(\tau - \frac{1-\exp(-k_i\tau)}{k_i} - \frac{1-\exp(-k_j\tau)}{k_j} + \frac{1-\exp(-(k_i+k_j)\tau)}{k_i+k_j}\right) \end{aligned} \tag{7}$$

Sometimes it is convenient to work with the equivalent annualized spot rate. From equation (5) we obtain

$$R(\mathbf{x}_t,\tau) = -\frac{1}{\tau}\log P(\mathbf{x}_t,\tau) = -\frac{1}{\tau}(\mathbf{u}(\tau)'\mathbf{x}_t + v(\tau)) \tag{8}$$

which is a linear function of the state variables. Therefore, under the generalized-Vasicek model, spot rates also have the Gaussian distribution.

The value of a coupon-bond $B(\mathbf{x}_t, t)$ with maturity $\tau = \tau_N$ and N coupons C_i paying at times τ_i can therefore be computed as

$$B(\mathbf{x}_t,\tau) = \sum_{i=1}^{N} C_i P(\mathbf{x}_t,\tau_i) \tag{9}$$

The implied yield to maturity of a coupon-bond maturing at τ, $y(\mathbf{x}_t,\tau)$, is obtained solving the following equation:

$$B(\mathbf{x}_t,\tau) = \sum_{i=1}^{N} C_i \exp(-y\tau_i) \tag{10}$$

Note that if $C_i \geqslant 0,\ \forall i \in [1, N]$, the relationship between $B(\mathbf{x}_t,\tau)$ and $y(\mathbf{x}_t,\tau)$ is one-to-one and continuous in the state variables. However, unlike spot rates, $y(\mathbf{x}_t,\tau)$ is not a linear function of the state variables and will not be normally distributed.

4. KALMAN FILTER ESTIMATION WITH INCOMPLETE PANEL-DATA

The Kalman filter is a widely used methodology which recursively calculates optimal estimates of unobservable state variables, given all the information available up to some moment in time. Using maximum likelihood methods, we can also obtain consistent estimates of model parameters. In finance, the Kalman filter has been used to estimate and implement stochastic models of interest rates,[6] commodities[7] and other relevant economic variables.[8]

In spite of its extensive use, the literature has not stressed on the Kalman filter's ability to use historical information when there are missing observations.[9] Most previous works have used complete panel-data, even at the cost of throwing away data on contracts not traded frequently or of aggregating data with close to, but not identical, maturities, with evident loss of information.[10] This problem is particularly acute in markets with infrequent trading where contracts with specific maturities do not trade every day. Below we show that a natural extension of the standard Kalman filter may be applied to jointly estimate the current term structure and its dynamics in markets with infrequent trading.

4.1. Standard Kalman filter

In this section we present a very brief description of the Kalman filter. For a detailed explanation, see, for example, Harvey (1989, Chapter 3) or Hamilton (1994, Chapter 13).

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The Kalman filter may be applied to dynamic models that are in a state-space representation, which include *measurement* and *transition* equations. At each point in time, the *measurement* equation relates a vector of observable variables $\mathbf{z}_t$ with a vector of state variables $\mathbf{x}_t$, which in general is not observable

$$\mathbf{z}_t = \mathbf{H}_t\mathbf{x}_t + \mathbf{d}_t + \mathbf{v}_t, \quad \mathbf{v}_t \sim N(\mathbf{0}, \mathbf{R}_t) \tag{11}$$

where $\mathbf{z}_t$ is a $m \times 1$ vector, $\mathbf{H}_t$ is a $m \times n$ matrix, $\mathbf{x}_t$ is a $n \times 1$ vector, $\mathbf{d}_t$ is a $m \times 1$ vector and $\mathbf{v}_t$ is a $m \times 1$ vector of serially uncorrelated Gaussian disturbances with mean $\mathbf{0}$ and covariance matrix $\mathbf{R}_t$. Even though we have implicitly assumed that vector $\mathbf{z}_t$ of observable variables is of a fixed size, we will later relax this assumption to allow for missing observations. Also, note that the measurement equation contains a disturbance term to allow for measurement errors in the observed data. Measurement equation (11) also assumes the existence of a linear relation between observed variables and state variables. This assumption will also be relaxed later on.

The *transition* equation describes the dynamics of the state variables

$$\mathbf{x}_t = \mathbf{A}_t\mathbf{x}_{t-1} + \mathbf{c}_t + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \mathbf{Q}_t) \tag{12}$$

where $\mathbf{A}_t$ is a $n \times n$ matrix, $\mathbf{c}_t$ is an $n \times 1$ vector and $\boldsymbol{\varepsilon}_t$ is an $n \times 1$ vector of serially uncorrelated Gaussian disturbances with mean $\mathbf{0}$ and covariance matrix $\mathbf{Q}_t$. Under this representation, the state variables have a multivariate normal distribution. This assumption can also be relaxed to include non-Gaussian models for the state variables. Equations (11) and (12) define what is called the state-space representation.[11]

The Kalman filter provides optimal estimates $\hat{\mathbf{x}}_t$ of the state variables given all the information up to time t. Let $\mathbf{P}_t$ be the covariance matrix of the estimation errors

$$\mathbf{P}_t = E(\mathbf{x}_t - \hat{\mathbf{x}}_t)(\mathbf{x}_t - \hat{\mathbf{x}}_t)^{\mathrm{T}} \tag{13}$$

Then, given $\hat{\mathbf{x}}_{t-1}$ and $\mathbf{P}_{t-1}$, which include all the information up to time $t-1$, the estimator of the state variables and the covariance matrix of the estimation errors at time t are

$$\hat{\mathbf{x}}_{t|t-1} = \mathbf{A}_t\hat{\mathbf{x}}_{t-1} + \mathbf{c}_t \tag{14}$$

$$\mathbf{P}_{t|t-1} = \mathbf{A}_t\mathbf{P}_{t-1}\mathbf{A}'_t + \mathbf{Q}_t \tag{15}$$

Equations (14) and (15) are usually called the *prediction* step.

When new information (represented by $\mathbf{z}_t$) becomes available, it is used to obtain an optimal estimate of the state variables and of the error covariance matrix

$$\hat{\mathbf{x}}_t = \hat{\mathbf{x}}_{t|t-1} + \mathbf{P}_{t|t-1}\mathbf{H}'_t\mathbf{F}_t^{-1}\mathbf{v}_t \tag{16}$$

$$\mathbf{P}_t = \mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\mathbf{H}'_t\mathbf{F}_t^{-1}\mathbf{H}_t\mathbf{P}_{t|t-1} \tag{17}$$

where

$$\mathbf{F}_t = \mathbf{H}_t\mathbf{P}_{t|t-1}\mathbf{H}'_t + \mathbf{R}_t \tag{18}$$

$$\mathbf{v}_t = \mathbf{z}_t - (\mathbf{H}_t\hat{\mathbf{x}}_{t|t-1} + \mathbf{d}_t) \tag{19}$$

Equations (16) and (17) correspond to what is usually called the *update* step.

Intuitively, the update step is just the calculation of the conditional expectation of state variables $\mathbf{x}_t$, given all the history of observations $\{\mathbf{z}_i\}_{i=1}^{i=t-1}$, and the new information $\mathbf{z}_t$, i.e. $\hat{\mathbf{x}}_t = E_{t-1}(\mathbf{x}_t|\mathbf{z}_t)$. It can be shown[12] that this conditional expectation is in fact an optimal estimation, in a mean square error sense, and corresponds to equation (16). The Kalman filter is thus a particular type of Bayesian estimation.

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Another useful characteristic of the Kalman filter, under the normality assumption, is that it provides consistent model parameters estimates $\hat{\boldsymbol{\psi}}$, when maximizing the log-likelihood function of error innovations

$$\log L(\boldsymbol{\psi}) = -\frac{1}{2}\sum_t \log |\mathbf{F}_t| - \frac{1}{2}\sum_t \mathbf{v}_t'\mathbf{F}_t^{-1}\mathbf{v}_t \tag{20}$$

where $\boldsymbol{\psi}$ represents a vector containing the unknown parameters.

Moreover, the covariance matrix of the estimation errors, $\mathbf{I}(\hat{\boldsymbol{\psi}})^{-1}$, may be obtained from the information matrix $\mathbf{I}(\boldsymbol{\psi})$

$$\mathbf{I}(\boldsymbol{\psi}) = \frac{\partial^2 \log L(\boldsymbol{\psi})}{\partial\boldsymbol{\psi}\partial\boldsymbol{\psi}'} \tag{21}$$

4.2. *Kalman filter applied to incomplete panel-data*

As already stated, existent literature stresses on the use of the Kalman filter methodology with complete panel-data sets. However, it is not necessary to assume a fixed number of observable variables at each time period in order to apply the Kalman filter.

Let m_t be the number of observations available at time t, which need not be equal to the number of observations available at any other date. This means that the number of observations available at any date is time dependent. The measurement equation is again

$$\mathbf{z}_t = \mathbf{H}_t\mathbf{x}_t + \mathbf{d}_t + \mathbf{v}_t, \quad \mathbf{v}_t \sim N(\mathbf{0}, \mathbf{R}_t) \tag{22}$$

but now $\mathbf{z}_t$ is a $m_t \times 1$ vector, $\mathbf{H}_t$ is a $m_t \times n$ matrix, $\mathbf{x}_t$ is a $n \times 1$ vector, $\mathbf{d}_t$ is a $m_t \times 1$ vector and $\mathbf{v}_t$ is a $m_t \times 1$ vector of serially uncorrelated Gaussian disturbances with mean $\mathbf{0}$ and covariance matrix $\mathbf{R}_t$ with dimension is $m_t \times m_t$. Under this assumptions, $\{\mathbf{z}_i\}_{i=1}^{i=T_N}$ will be considered an incomplete panel-data set.

To see why the Kalman filter still may be used with incomplete panel-data sets, note that given a vector of state variables $\hat{\mathbf{x}}_{t-1}$ and a covariance matrix $\mathbf{P}_{t-1}$ of the estimation errors, the filter first calculates a prediction of the state variables $\hat{\mathbf{x}}_{t|t-1}$ and of the covariance matrix $\mathbf{P}_{t|t-1}$ of the errors using equations (14) and (15). For this calculation only the dynamic properties of the state variables are used which do not depend on the number of observable variables.

The filter then incorporates the new information given by the vector of observable variables $\mathbf{z}_t$. The same equations (16) and (17) can then be used to calculate optimal estimates of the state vector $\hat{\mathbf{x}}_t$ and of the covariance matrix $\mathbf{P}_t$. As mentioned before, since the Kalman filter computes at every date the conditional expectation $\hat{\mathbf{x}}_t = E_{t-1}(\mathbf{x}_t|\mathbf{z}_t)$, the estimates can still be computed, even if the number of observations vary with time. Of course, the greater the number of observations available to update the filter, the better the accuracy of the estimation. This is reflected in a lower variance of the estimation error.

When a reduced number of observations is available at some date, the estimation error and its variance will be greater, reflecting more uncertainty on the true value of the state variables. In any case, the estimation of the state variables takes into account the whole variance–covariance structure among observations.

4.3. *Kalman filter with a non-linear measurement equation*

When applying the Kalman filter to coupon-bond yields (or prices), we usually obtain a non-linear measurement equation. In this case the extended Kalman filter, which applies to non-linear measurement and/or transition equations, must be used. We will briefly[13] describe the mathematics of the extended Kalman filter.

Since under the generalized-Vasicek model, which has been used to illustrate the methodology, the transition equation is a linear function of the state variables, we restrict the analysis to the case where only the measurement equation is a non-linear function of the state variables.[14]

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Let the measurement equation be a non-linear function of the state variables

$$\mathbf{z}_t = \mathbf{f}_t(\mathbf{x}_t) + \mathbf{v}_t, \quad \mathbf{v}_t \sim N(\mathbf{0}, \mathbf{R}_t) \tag{23}$$

with $\mathbf{f}_t : \Re^n \rightarrow \Re^{m_t}$ a continuous and differentiable function.[15]

The extended Kalman filter, when only the measurement equation is non-linear, is obtained by linearizing $\mathbf{f}_t(\mathbf{x}_t)$ around the conditional mean $\hat{\mathbf{x}}_{t|t-1}$

$$\mathbf{f}_t(\mathbf{x}_t) = \mathbf{f}_t(\hat{\mathbf{x}}_{t|t-1}) + \overline{\mathbf{H}}_t(\mathbf{x}_t - \hat{\mathbf{x}}_{t|t-1}) \tag{24}$$

where $\bar{\mathbf{H}}_t = (\partial/\partial \mathbf{x}_t')\mathbf{f}_t(\mathbf{x}_t)\big|_{\mathbf{x}_t=\hat{\mathbf{x}}_{t|t-1}}$

The prediction step equations are the same as before. The update step equation under the extended Kalman filter is then

$$\hat{\mathbf{x}}_t = \hat{\mathbf{x}}_{t|t-1} + \mathbf{P}_{t|t-1}\overline{\mathbf{H}}_t'\mathbf{F}_t^{-1}\mathbf{v}_t \tag{25}$$

$$\mathbf{P}_t = \mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\overline{\mathbf{H}}_t'\mathbf{F}_t^{-1}\overline{\mathbf{H}}_t\mathbf{P}_{t|t-1} \tag{26}$$

where

$$\mathbf{F}_t = \overline{\mathbf{H}}_t\mathbf{P}_{t|t-1}\overline{\mathbf{H}}_t' + \mathbf{R}_t \tag{27}$$

$$\mathbf{v}_t = \mathbf{z}_t - \mathbf{f}_t(\hat{\mathbf{x}}_{t|t-1}) \tag{28}$$

An explanation on how to apply the extended Kalman filter to coupon-bond yields can be found in the Appendix.

5. EMPIRICAL RESULTS

To illustrate our methodology, we estimate a three-factor generalized-Vasicek model using Chilean government bond data.[16] The data used consist of inflation-protected bonds, the most liquid fixed-income instrument traded in Chile. Thus, we are modelling the behaviour of real, as opposed to nominal, interest rates. The choice of the Vasicek model seems appropriate for modelling real rates which might become negative whenever the rate of inflation exceeds the nominal interest rate.

Given that most of the outstanding bonds trade only sporadically, the Chilean government bond market can be characterized as a market with infrequent trading and is used to test our term-structure estimation methodology.

In the following sections we describe the data and analyse the estimation results based on in-sample and out-of-sample yield errors and on the ability of the model to fit the observed term-structure of volatilities.

5.1. Data description

The data consist of all transactions at the Santiago Stock Exchange from January 1997 to December 2001 (1243 days) of pure-discount bonds and semi-annual amortizing coupon bonds issued by the Chilean government. Pure-discount bonds are usually denominated 'Pagare Reajustable Banco Central' (PRBC) bonds, and semi-annual amortizing coupon bonds are called 'Pagare Reajustable con Cupones' (PRC) bonds. Both type of bonds are inflation-protected with payments brought to real terms using monthly inflation.[17]

Table 1 summarizes the data. It can be noted that pure-discount bonds have maturities of less than 1 year while coupon bonds have maturities ranging from 1 to 20 years. Trading frequency is defined as the number of days for which we have at least one transaction of a bond of a specific maturity over all available trading days. A trading frequency of 20% means that at least one bond with that maturity was traded an average of 50 days per year. From Table 1 we see that for most maturities, the trading frequency ranges from 30% to

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Table 1. Description of the data: daily transactions of Chilean government inflation-protected pure discount and coupon bonds from January 1997 to December 2001

Maturity range (Years)	Number of observations	Average trading frequency[a] (%)	Average yield[b] (%)	Yield standard deviation[b] (%)
Pure discount bonds				
0–1	1115	89.70	5.81	2.04
Coupon bonds				
1–1.5	377	30.33	6.46	1.83
1.5–2.5	426	34.27	6.29	1.45
2.5–3.5	443	35.64	6.20	1.17
3.5–4.5	642	51.65	6.15	1.17
4.5–5.5	519	41.75	6.36	1.12
5.5–6.5	550	44.25	6.36	0.87
6.5–7.5	766	61.63	6.33	0.91
7.5–8.5	921	74.09	6.22	0.81
8.5–9.5	451	36.28	6.31	0.80
9.5–10.5	584	46.98	6.31	0.65
10.5–11.5	268	21.56	6.30	0.72
11.5–12.5	458	36.85	6.21	0.67
12.5–13.5	262	21.08	6.20	0.64
13.5–14.5	507	40.79	6.14	0.60
14.5–15.5	269	21.64	6.10	0.71
15.5–16.5	311	25.02	6.13	0.61
16.5–17.5	269	21.64	6.18	0.60
17.5–18.5	309	24.86	6.32	0.53
18.5–19.5	404	32.50	6.32	0.53
19.5–20	533	42.88	6.26	0.60
Total	10 384			

[a] Trading frequency is defined as the number of days for which there is a transaction of a given bond over all available trading days.
[b] Continuous compounding.

45%. Standard deviation of observed yields generally decreases as bond maturity increases, which is consistent with mean reversion in interest rates.

Figure 4 illustrates the sparseness or infrequent trading of daily bond transactions in Chile by showing for each day during the second semester of 2001 when a bond was traded or not. The panel-data shown are clearly incomplete, a condition that is critical in the choice of the estimation methodology.[18]

5.2. *Estimation results*

We estimate the three-factor Vasicek model parameters using bond price transactions data from January 1997 to December 2001. As noted in Section 4, the Kalman filter considers measurement errors in the observations. For simplicity we assume that the error variance–covariance matrix $\mathbf{R}_t$ is diagonal. Also, we aggregate bonds into five groups depending on their maturities: the first group includes the discount bonds with maturities up to 1 year, and the next four groups include coupon bonds with maturities ranging from 1 to 5 years, from 6 to 10 years, from 11 to 15 years and from 16 to 20 years, respectively. Bonds within each group are assumed to have measurement errors with the same standard deviation: ξ^d, ξ_1^c, ξ_2^c, ξ_3^c and ξ_4^c, respectively. With these assumptions 18 different parameters must be estimated.[19] Table 2 presents parameter estimates and their respective estimation errors. Note that all the parameters are statistically significant, though the mean reversion coefficient of the first factor is very small suggesting that this factor follows a process which is close to a random walk.

Note that the correlation between the factors is very high which may lead us to believe that two factors could be sufficient to explain the dynamics of the yield curve. However, we find that with one and two factors the total in-sample RMSE is 0.52% and 0.35%, respectively, compared with 0.12% obtained using

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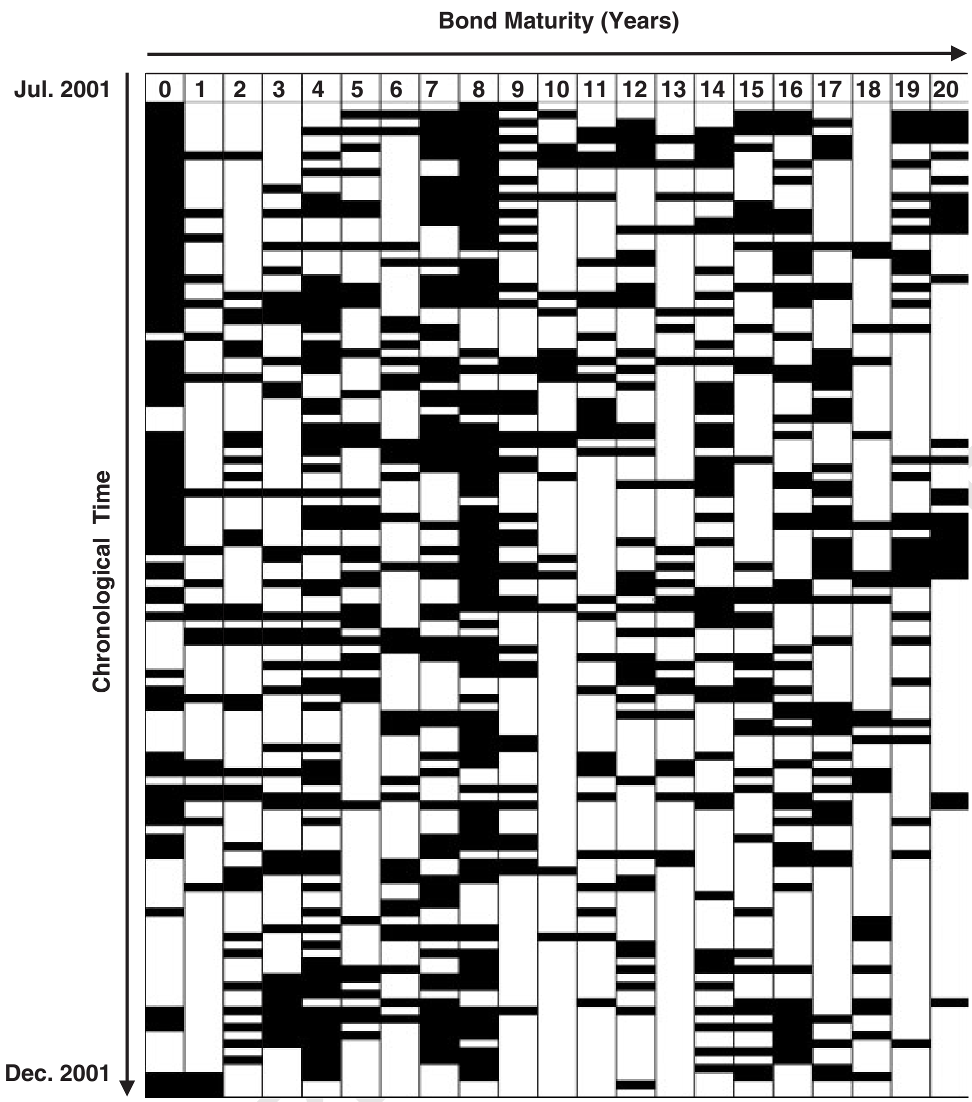

Figure 4. Graphical description of available Chilean government inflation-protected discount and coupon bond daily data for the second semester of 2001. A black cell indicates that data were available for the corresponding maturity at a given day.

three factors. Therefore, this important difference in estimation errors suggests that a three-factor model is necessary to explain the complex dynamics of the Chilean yield curve.

To illustrate the ability of the approach to fit observed prices on a day with a large number of transactions, Figure 5 shows the yield curve derived from the model for 01/09/1997. We see that the model is able to fit very well observed yields and this is representative of the sample period.

Recall that in Figure 2 we illustrated the inability of the curve-fitting methods to provide for reliable long-term rates for a day when only short-term bonds were traded. Figure 6 shows the yield curve obtained for the same day (10/06/1999) using our proposed methodology. We see that the estimated yield curve not only correctly fits observed yields for that day, but also is consistent with the previous day observations. Note that the yield curve shown has been constructed using only prices for that particular day, and the dynamics of the interest rate process. We have not included the previous day curve in Figure 6 because it is almost identical to the curve shown. The model's long-term yields for the current day, for which there is no data, are very close to the observed previous day long-term yields. Comparing Figure 6 with Figure 2 which

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Table 2. Parameter estimates and standard errors from daily transactions of Chilean government inflation-protected pure discount and coupon bonds from January 1997 to December 2001

κ_1	0.00050	0.00012
κ_2	1.11455	0.01681
κ_3	2.16431	0.05362
σ_1	0.01747	0.00019
σ_2	0.29298	0.00466
σ_3	0.32780	0.00647
ρ_{21}	−0.91042	0.01258
ρ_{31}	0.84189	0.02376
ρ_{32}	−0.97121	0.00246
λ_1	−0.00056	0.00002
λ_2	0.01599	0.00418
λ_3	−0.05213	0.01836
δ	0.05614	0.02654
ξ^d	0.00225	0.00014
ξ_1^c	0.00225	0.00004
ξ_2^c	0.00079	0.00001
ξ_3^c	0.00027	0.00001
ξ_4^c	0.00038	0.00001

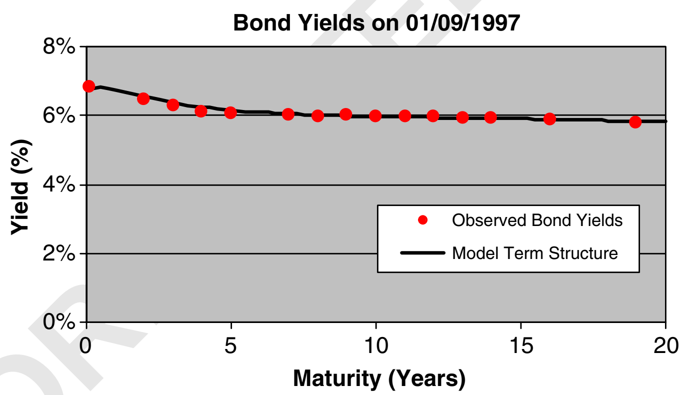

Figure 5. Estimated and observed coupon-bond-yields on 01/09/1997.

corresponds to the same date, this example illustrates that our approach provides much more stable curves than those obtained by curve-fitting methods.

Table 3 presents in-sample and out-of-sample error measures by maturity. Out-of-sample error measures were calculated by re-estimating the model using data from 1997 to 2000, and then comparing yield curves obtained from the model to observed yields for the year 2001, which was not used in the parameter estimation. It can be seen that all errors are reasonably low, while errors for short-term bonds are larger than for long-term bonds. Out-of-sample errors are similar to in-sample errors, showing the stability of the model and its ability to be used in real-world applications.

Finally, we analyse the volatility term structure of spot interest rates and compare it to volatilities obtained directly from bond yields. The theoretical volatility structure of interest rates, which is

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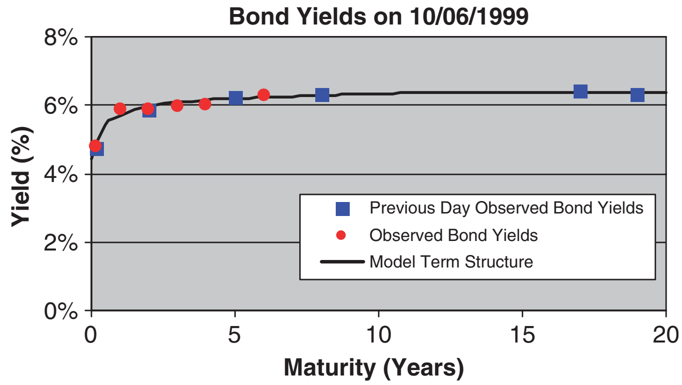

Figure 6. Estimated and observed coupon-bond-yields on 10/06/1999.

Table 3. In-sample and out-of-sample RMSE for the year 2001

Maturity range (Years)	RMSE in-sample (%)	RMSE out-of-sample (%)
Discount bonds		
0–1	0.14	0.12
Coupon bonds		
1–1.5	0.25	0.33
1.5–2.5	0.16	0.23
2.5–3.5	0.17	0.21
3.5–4.5	0.13	0.15
4.5–5.5	0.16	0.16
5.5–6.5	0.06	0.06
6.5–7.5	0.05	0.06
7.5–8.5	0.06	0.09
8.5–9.5	0.06	0.08
9.5–10.5	0.05	0.06
10.5–11.5	0.04	0.04
11.5–12.5	0.03	0.03
12.5–13.5	0.03	0.03
13.5–14.5	0.03	0.02
14.5–15.5	0.02	0.02
15.5–16.5	0.03	0.03
16.5–17.5	0.03	0.03
17.5–18.5	0.03	0.03
18.5–19.5	0.03	0.04
19.5–20	0.03	0.04
Total		
0–20	0.10	0.11

independent of the state variables, is obtained by applying Ito's lemma to equation (8)

$$\sigma_R(\tau) = \left(\sum_{i=1}^{N} \sum_{j=1}^{N} u_i(\tau) u_j(\tau) \sigma_i \sigma_j \rho_{ij} \right)^{1/2} \tag{29}$$

where

$$u_i(\tau) = -\frac{1 - \exp(-k_i \tau)}{k_i} \tag{30}$$

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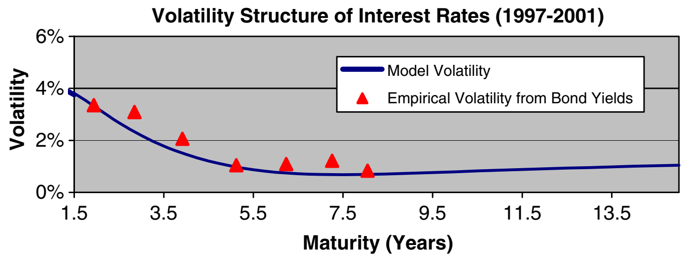

Figure 7. Volatility structure of interest rates 1997–2001.

There are two difficulties in computing empirical estimates of the interest rate volatilities. First, most of the data consist of amortizing coupon bonds and we are interested in the volatility of spot rates. Second, the panel-data contain many missing observations. To address these problems we aggregate the data in groups according to their maturity. The first group contains bonds with one to two years of maturity, and so on. Then, for each date we take the average yield of all the bonds in a given group and we compute the volatility of daily changes of these yields. In addition, we compute the average duration of the bonds in each group. To compare this empirical volatility to model spot volatilities, we assume that the volatility of each group represents the volatility of a discount bond with maturity equal to the average duration in the group.

Figure 7 shows the term structure of spot volatilities from the model and from the empirical estimates. Comparing this figure with Figure 3, we observe that our model volatilities are much closer to the empirical volatilities than those obtained using the curve-fitting methods.

6. CONCLUSION

The estimation of the term structure of interest rates is a critical issue, not only from a theoretical point of view, but also for all market participants including banks, regulators and financial institutions. It is an essential ingredient in the valuation and hedging of all fixed-income securities. It is also necessary for financial planning and for implementing monetary policy. In economies with well-developed and liquid financial markets, the existence of bond prices for a wide range of different maturities makes it easy to extract a term structure of spot rates that explains observed prices. Moreover, in some countries, such as the United States, zero-coupon bonds (Strips) of different maturities are individually traded. In many emerging markets, however, bonds trade infrequently so that for every particular day there are bond prices for only a few maturities. This missing-observation problem makes it difficult, and sometimes impossible, to estimate the term structure using only current data.

In this article we develop a methodology for using an incomplete panel-data of bond price observations to estimate the current term structure. We use an extended Kalman filter approach to estimate a dynamic multi-factor model of interest rates using the panel-data with missing observations. The Kalman filter estimation provides not only the parameters of the model but also the time-series of the factors.

The approach jointly estimates the current term structure and its dynamics. The model can be used to value and hedge all types of interest rate derivatives, including bonds with embedded options. This methodology also allows us to estimate the term structure for days with an arbitrary small number of traded bonds.

We implement the approach using a three-factor generalized-Vasicek (1977) model and Chilean government bond data. The methodology, however, can be implemented with a broad class of dynamic interest rate models and in any market with infrequent trading, a very common situation in many emerging markets.

Our approach is currently being used by a consortium of financial and academic institutions in Chile to estimate the Chilean term structure of interest rates. The results are updated daily at the website RiskAmerica.com.

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APPENDIX

In this appendix we describe in detail how to apply the methodology developed in Section 4 to the generalized-Vasicek model introduced in Section 3, with an incomplete panel-data set of discount and coupon-bond-yields.

The transition equation of the state variables under a generalized-Vasicek model is independent of the observations and the associated terms appearing in equation (12) are

$$\mathbf{A}_t = \mathrm{diag}_n(1 - k_i \Delta t), \quad \mathbf{c}_t = \begin{pmatrix} -\lambda_1 \Delta t \\ \vdots \\ -\lambda_n \Delta t \end{pmatrix}, \quad \mathbf{Q}_t = \begin{pmatrix} \sigma_1^2 & \cdots & \sigma_1 \sigma_n \rho_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_n \sigma_1 \rho_{n1} & \cdots & \sigma_n^2 \end{pmatrix} \Delta t \tag{A1}$$

where $\mathrm{diag}_n(x_i)$ stands for a diagonal $n \times n$ matrix whose (i, i) element is x_i, Δt is the time interval at which yields are observed and other parameters are the ones appearing in equation (4).

Let m_t^d and m_t^c be the number at time t of observed discount and coupon bonds, respectively, and $\{\tau_{i,t}^d\}_{i=1}^{n_t^d}$ and $\{\tau_{i,t}^c\}_{i=1}^{n_t^c}$ the sets containing their respective associated maturities. The vector of observations $\mathbf{z}_t$ is then

$$\mathbf{z}_t = \begin{pmatrix} \mathbf{z}_t^d \\ \mathbf{z}_t^c \end{pmatrix} \tag{A2}$$

where $\mathbf{z}_t^d$ and $\mathbf{z}_t^c$ are $m_t^d \times 1$ and $m_t^c \times 1$ vectors containing the observed yields of discount and coupon bonds, respectively. Of course, either m_t^d or m_t^c can be zero, but not both at the same time. The parameters of the measurement equation are

$$\mathbf{H}_t = \begin{pmatrix} \mathbf{H}_t^d \\ \mathbf{H}_t^c \end{pmatrix}, \quad \mathbf{d}_t = \begin{pmatrix} \mathbf{d}_t^d \\ \mathbf{d}_t^c \end{pmatrix} \tag{A3}$$

$$\mathbf{H}_t^d = \begin{pmatrix} -\dfrac{\mathbf{u}(\tau_{1,t}^d)'}{\tau_{1,t}^d} \\ \vdots \\ -\dfrac{\mathbf{u}(\tau_{m_t^d,t}^d)'}{\tau_{m_t^d,t}^d} \end{pmatrix}, \quad \mathbf{H}_t^c = \begin{pmatrix} \dfrac{\partial}{\partial \mathbf{x}'} y(\hat{\mathbf{x}}_{t|t-1}, \tau_{1,t}^c) \\ \vdots \\ \dfrac{\partial}{\partial \mathbf{x}'} y(\hat{\mathbf{x}}_{t|t-1}, \tau_{m_t^c,t}^c) \end{pmatrix} \tag{A4}$$

$$\mathbf{d}_t^d = \begin{pmatrix} -v(\tau_{1,t}^d) \\ \vdots \\ -v(\tau_{m_t^d,t}^d) \end{pmatrix}, \quad \mathbf{d}_t^c = \begin{pmatrix} y(\hat{\mathbf{x}}_{t|t-1}, \tau_{1,t}^c) - \left(\dfrac{\partial}{\partial \mathbf{x}'} y(\hat{\mathbf{x}}_{t|t-1}, \tau_{1,t}^c)\right) \hat{\mathbf{x}}_{t|t-1} \\ \vdots \\ y(\hat{\mathbf{x}}_{t|t-1}, \tau_{m_t^c,t}^c) - \left(\dfrac{\partial}{\partial \mathbf{x}'} y(\hat{\mathbf{x}}_{t|t-1}, \tau_{m_t^c,t}^c)\right) \hat{\mathbf{x}}_{t|t-1} \end{pmatrix} \tag{A5}$$

The gradient of the yield with respect to state variables can be computed by differentiating implicitly equation (10) with respect to the state variables

$$\begin{aligned} \frac{\partial}{\partial \mathbf{x}} \left(\sum_{j=1}^{M} \exp\left(\mathbf{u}(\tau_j)^{\mathrm{T}} \mathbf{x} + v(\tau_j)\right) \right) &= \frac{\partial}{\partial \mathbf{x}} \left(\sum_{j=1}^{M} \exp\left(-y(\mathbf{x}, \tau)\tau_j\right) \right) \\ &= \frac{\partial}{\partial y} \left(\sum_{j=1}^{M} \exp\left(-y(\mathbf{x}, \tau)\tau_j\right) \right) \frac{\partial y(\mathbf{x}, \tau)}{\partial \mathbf{x}} \end{aligned} \tag{A6}$$

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so that

$$\frac{\partial y(\mathbf{x},\tau)}{\partial \mathbf{x}} = \frac{\sum_{j=1}^{M} \mathbf{u}(\tau_j)\exp(\mathbf{u}(\tau_j)^{\mathrm{T}}\mathbf{x} + v(\tau_j))}{\sum_{j=1}^{M} -\tau_j \exp(-y(\mathbf{x},\tau)\tau_j)} \tag{A7}$$

The remaining parameters to be specified belong to the covariance matrix of measurement errors. In this paper, we assume that this covariance matrix is diagonal and can only have five different parameters: ξ^d, ξ_1^c, ξ_2^c, ξ_3^c and ξ_4^c. The first of them corresponds to the variance of measurement errors of discount bonds. The remaining four parameters correspond to the variance of coupon bonds for maturities ranging between 1 to 5 years, 6 to 10 years, 11 to 15 years and 16 to 20 years, respectively. Therefore, the covariance matrix of measurement errors is

$$\mathbf{R}_t = \begin{pmatrix} \mathbf{R}_t^d & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_t^c \end{pmatrix} \tag{A8}$$

where $\mathbf{R}_t^d = \mathrm{diag}_{n_t^d}(\boldsymbol{\xi}^d)$ and $\mathbf{R}_t^c = \mathrm{diag}_{n_t^c}(\boldsymbol{\xi}_j^c)$ are diagonal matrices.

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NOTES

1. See the working paper version of this article for details on these methods.
2. The coupon bonds considered here are amortizing bonds paying semi-annually equal coupons. These instruments are described in more detail in Section 5.
3. In a mean reverting model, every perturbation is on average reduced by half in $\log(2)/k_i$ units of time.
4. The canonical form proposed by Dai and Singleton (2000) for Gaussian interest rates allows for the possibility of common eigenvalues in matrix **K**. To obtain simpler analytical formulas for the prices of pure discount bonds, we impose the condition that all eigenvalues are different, but this restriction may easily be relaxed.
5. We assume for simplicity that risk premiums are constant, but this could be extended to any linear function of the state variables.
6. For example see Lund (1994, 1997), Duan and Simonato (1999), Geyer and Pichler (1999), Babbs and Nowman (1999), de Jong and Santa-Clara (1999) and de Jong (2000).
7. For example see Schwartz (1997), Schwartz and Smith (2000) and Sørensen (2002).
8. See, for example, Pennacchi (1991) and Dewachter and Maes (2001).
9. An exception is Sørensen (2002) who has applied Kalman filter for incomplete panel-data in the commodity markets.
10. Cortazar and Schwartz (2003) discuss this issue and propose an alternative approach that does not use the Kalman filter to deal with this problem of missing observations and apply it to commodity futures.
11. The state-space representation of the generalized-Vasicek model is described in the Appendix.
12. See, for example, Øksendal (1998).
13. Additional information can be found in Harvey (1989).
14. For example, under a CIR model, the resulting transition equation is also nonlinear. See Lund (1994, 1997), Duan and Simonato (1999), Geyer and Pichler (1999) and Chen and Scott (2003).
15. In this analysis we assume the general case of an incomplete panel-data setting, hence the dimension of the function range depends on the number of observations available at time t. In a complete panel-data setting, this time dependence disappears.
16. These instruments are actually issued by the Chilean Central Bank, an institution equivalent to the Federal Reserve in the US.
17. In practice this is done by expressing payments in another unit, the UF ('Unidad de Fomento'), which is updated every month using the previous month inflation.
18. Curiously, the figure resembles a DNA pattern.
19. Implementation issues of the model can be found in the Appendix.

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AN N-FACTOR GAUSSIAN MODEL OF OIL FUTURES PRICES

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This article studies the ability of an N-factor Gaussian model to explain the stochastic behavior of oil futures prices when estimated with the use of all available price information, as opposed to traditional approaches of aggregating data for a set of maturities. A Kalman filter estimation procedure that allows for a time-dependent number of daily observations is used to calibrate the model. When applied to all daily oil futures price transactions from 1992 to 2001, the model performs very well, requiring at least three factors to explain the term structure of futures prices, but four factors to fit the volatility term structure. The model also performs very well for daily copper futures transactions from 1992 to 2001 and for out-of-sample daily oil futures transactions from 2002 to 2004. © 2006 Wiley Periodicals, Inc. Jrl Fut Mark 26:243–268, 2006

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INTRODUCTION

The valuation and hedging of commodity contingent claims has received a great amount of attention by both academics and practitioners, and has become an important area of financial economics. Inextricably interwoven with this issue is the modeling and estimation of the stochastic behavior of commodity prices. The practical implication of having better models and estimation methodologies is that commodity producers and consumers, and also financial intermediaries, may implement sound investment and risk-management strategies with vast economic implications. On the contrary, the application of naïve models can lead to unreliable results, which may include heavy financial losses to corporations (Culp & Miller, 1994).

Among commodities traded in financial markets, oil is one of the most important, and has been studied in the literature extensively. Its relevance induces innovations in financial markets, generating new oil contingent claims that should be used in the estimation of the stochastic behavior of oil prices.

One important source of information for the study of oil prices is the futures market. Oil futures markets have included in recent years new futures contracts with longer maturities up to 7 years. When new contracts are introduced there is no historical information. In addition, not all futures contracts trade every day. If a complete data set of prices is to be used, some prices need to be discarded or aggregated, with great information loss. Thus, it would be desirable that the estimation procedure uses all price information available.

Oil prices are very volatile, have a high degree of mean reversion (Bessembinder, Coughenour, Seguin, & Smoller, 1995), and exhibit complex dynamics. Thus, it is important to analyze the number of risk factors required to model this stochastic behavior (Cortazar & Schwartz, 1994).

Several models of the stochastic process followed by commodity prices have been proposed in the literature. They differ in how they specify spot price innovations and how they model the cost of carry. The cost of carry represents the storage cost plus the interest paid to finance the asset minus the net benefit that accrues to the asset holder, if any. In the commodities literature, the benefit received by the commodity owner, but not by the futures contract owner, is called the *convenience yield* (Brennan, 1958, 1991; Deaton & Laroque, 1992; Gibson & Schwartz, 1990; Routledge, Seppi, & Spatt, 2000; Working, 1949), which is commonly represented as a dividend yield.

Early models of commodity prices assume a one-factor geometric Brownian motion for the spot price with a constant interest rate and convenience yield, which implies a constant cost of carry (Brennan & Schwartz, 1985). Even though this widely used and simple model has the advantage of being very tractable, it has some undesirable properties like exhibiting a constant volatility term structure of futures price returns. Empirical evidence suggests, however, that the volatility term structure of futures prices is a decreasing function of maturity (Bessembinder, Coughenour, Seguin, & Smoller, 1996), which may be explained by the existence of mean reversion in commodity prices (Bessembinder et al., 1995).

To address this issue, several authors have proposed different one-factor models that take into account mean reversion in commodity prices (Laughton & Jacoby, 1993, 1995; Ross, 1997; Schwartz, 1997). However, an empirical implication of all models that consider a single source of uncertainty is that futures prices for different maturities should be perfectly correlated, which defies existing evidence.

To account for a more realistic model of commodity prices, two- and three-factor models have been proposed[1] (Cortazar & Schwartz, 2003; Gibson & Schwartz, 1990; Hilliard & Reis, 1998; Schwartz, 1997; Schwartz & Smith, 2000). The advantage of using more factors in modeling the spot price process and the cost of carry is that a better fit to observed futures prices may be obtained. This goodness of fit can generally be observed not only in terms of mean-squared errors, but also by comparing empirical and model-implied volatility term structures, which is critical for valuing option-like contingent claims and also for risk-management applications.

In addition to defining the commodity price model, a methodology to obtain parameter estimates must be chosen. For Gaussian models such as the one presented in this article, a closed-form formula of the probability distribution of futures prices is known, and parameters may be obtained by maximizing their likelihood function. Therefore, consistent estimates of model parameters are obtained with their respective estimation errors.

Some difficulties must be addressed, however, to successfully apply these models to commodity markets. For example, most multifactor models are based on nonobservable state variables that must be estimated from observed prices. Thus, in addition to calibrating model parameters,

[1]A different approach for modeling the spot price of a commodity is based on the Heath, Jarrow, and Morton (1992) no-arbitrage model (Cortazar & Schwartz, 1994; Miltersen & Schwartz, 1998).

which are assumed constant for each data set, it is necessary to estimate state variables for each date.

When state variables and futures prices are related by a closed-form formula, it is possible to estimate the unobserved state variables by inverting the pricing formula (Chen & Scott, 1993; Duffie & Singleton, 1997; Pearson & Sun, 1994). In this case, it is assumed that futures prices are observed without measurement error. However, for any given date the number of available prices is generally higher than the number of state variables that need to be estimated. Therefore, it must be assumed that observed prices have some degree of measurement error, which has to be assigned across the different contracts. Moreover, it may be desirable to include as many observed futures prices as possible in the estimation process of the state variables.

One of the most successful econometric procedures that takes into account the above issues is the Kalman filter, a widely used estimation methodology that can handle multifactor models with nonobservable state variables and measurement errors. In addition, it is capable of using a large price panel in the estimation process, avoiding the necessity of making an arbitrary selection of contracts to include in the estimation. The Kalman filter has been used in finance to estimate state variables of commodity price models by Schwartz (1997), Schwartz and Smith (2000), Manoliu and Tompaidis (2002), and Sørensen (2002), among others.

Traditional implementations of the Kalman filter normally assume a complete panel data set. This implies that for all given dates in the estimation sample, prices for the same set of contracts (with the same maturities) must be observed. This is not normally the case, because financial markets have innovations, and new contracts with longer maturities are frequently introduced. Traditional applications of the Kalman filter typically address this missing-data problem by aggregating or discarding data, with the consequent loss of information.

The missing-data problem may be so relevant that some authors have chosen not to use the Kalman filter, but to propose an alternative procedure to handle cases where the panel data are incomplete. Cortazar and Schwartz (2003) propose a very simple estimation procedure and apply it to an incomplete panel of oil futures prices. The methodology, however, does not make an optimal use of prices in the estimation of state variables (as opposed to the Kalman filter), and is unable to obtain parameter estimation errors.

An alternative procedure used in this article is to modify the traditional application of the Kalman filter to address incomplete panel-data

conditions. Sørensen (2002) uses this procedure for a seasonal price model, and Cortazar, Schwartz, and Naranjo (2003) use it for estimating the term structure of interest rates in an emerging market with low-frequency transactions. However, this approach has not received much attention in the literature.

This article studies the ability of an N-factor Gaussian model to explain the stochastic behavior of oil futures prices when estimated with the use of all price information available, as opposed to traditional approaches of aggregating data for a set of maturities. A Kalman filter estimation procedure that allows for a time-dependent number of daily observations is used to calibrate the model. When applied to all daily oil futures price transactions from 1992 to 2001, the model performs very well, requiring at least three factors to explain the term structure of futures prices, but four factors to fit the volatility term structure.

The organization of the article is as follows. The next section explains the N-factor Gaussian model for the spot price of oil. The Kalman filter methodology is then presented in an incomplete panel-data setting. Estimation results for oil futures prices are presented, and a conclusion is provided.

OIL PRICE MODEL AND FUTURES VALUATION

In this section an N-factor Gaussian model for the spot price of a commodity is presented, as well as its relation with other models commonly found in the commodities literature. In addition, valuation formulas for futures contracts and the theoretical volatility term structure of futures returns are obtained.

The Model

The N-factor model presented in this article generalizes existing two- and three-factor models commonly found in the literature (Cortazar & Schwartz, 2003; Gibson & Schwartz, 1990; Schwartz, 1997; Schwartz & Smith, 2000) to an N-factor setting. The model is based on the $A_0(N)$ canonical representation of Dai and Singleton (2000) for interest rates, which can be traced back to Vasicek (1977) and Langetieg (1980). However, in contrast to the interest-rates literature, which usually assumes a stationary process for the underlying spot rate, the Gaussian model for the spot price presented in this article is nonstationary, as it is usually assumed in the commodities literature.

Even though one-factor models may be able to explain a sizable fraction of the total price variance, these models tend to fit observed futures prices and the term structure of the volatility of futures returns rather poorly. The optimal number of factors that should be specified in a model depends on the stochastic behavior of the term structure of the specific commodity that is being modeled (Cortazar & Schwartz, 1994), and on the complexity that the modeler is willing to accept.

The N-factor model presented in the following extends existing models of commodity prices to an arbitrary number of factors while providing simple analytic valuation formulas for futures prices. This renders the model tractable and easy to implement and calibrate. Moreover, the model is Gaussian, which allows the use of the Kalman filter to estimate unobserved state variables and the use of maximum-likelihood techniques to calibrate model parameters. However, previous commodity literature has only focused in using one-, two-, and three-factor models, without extending its use to a general N-factor setting.

In this model, the spot price process of the commodity can be described as

$$\log S_t = \mathbf{1}'\mathbf{x}_t + \mu t \tag{1}$$

where $\mathbf{x}_t$ is a $n \times 1$ vector of state variables and μ, the long-term growth rate, is a constant. The vector of state variables $\mathbf{x}_t$ follows the process

$$d\mathbf{x}_t = -\mathbf{K}\mathbf{x}_t\,dt + \mathbf{\Sigma}\,d\mathbf{w}_t \tag{2}$$

where

$$\mathbf{K} = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 0 & \kappa_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \kappa_n \end{pmatrix} \quad \text{and} \quad \mathbf{\Sigma} = \begin{pmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_n \end{pmatrix}$$

are $n \times n$ diagonal matrices with entries that are positive constants. Also, $d\mathbf{w}_t$ is a $n \times 1$ vector of correlated Brownian motion increments such that $(d\mathbf{w}_t)'(d\mathbf{w}_t) = \mathbf{\Omega}\,dt$, where the (i, j) element of $\mathbf{\Omega}$ is $\rho_{ij} \in [1, -1]$, the instantaneous correlation between state variables i and j.

This model specification implies that the state variables have a multivariate Normal distribution. The first state variable follows a random walk, inducing a unit root in the spot price process. Each of the other state variables reverts to zero at a mean reversion rate given by k_i. In

order to compare the present model to traditional models found in the commodities literature, κ_1 has been exogenously set to zero.[2]

By assuming a constant risk premium[3] $\boldsymbol{\lambda}$, the risk-adjusted process for the vector of state variables is

$$d\mathbf{x}_t = -(\boldsymbol{\lambda} + \mathbf{K}\mathbf{x}_t)dt + \boldsymbol{\Sigma}\, d\mathbf{w}_t^* \tag{3}$$

where $\boldsymbol{\lambda}$ is a $n \times 1$ vector of real constants.

Instead of modeling the risk-free interest rate and the convenience yield independently (Gibson & Schwartz, 1990), this article models the cost of carry c_t (Schwartz & Smith, 2000), defined as the difference between the instantaneous risk-free interest rate r_t and the convenience yield δ_t.

Affine Transformations of the Model

This subsection shows how to rewrite a model through an affine transformation, and this procedure is later applied to the Gibson and Schwartz (1990) model to show that this is a particular case of the N-factor model presented in this article.

Consider the following model:

$$\log S_t = \bar{\mathbf{h}}'\boldsymbol{\zeta}_t + \bar{\mu}(t) \tag{4}$$

$$d\boldsymbol{\zeta}_t = (-\bar{\mathbf{K}}\boldsymbol{\zeta}_t + \bar{\boldsymbol{\beta}})\,dt + \bar{\boldsymbol{\Sigma}}\, d\bar{\mathbf{w}}_t \tag{5}$$

where $\boldsymbol{\zeta}_t$ is a vector of state variables, $d\bar{\mathbf{w}}_t d\bar{\mathbf{w}}_t' = \bar{\boldsymbol{\Omega}}\,dt$, $\bar{\mu}(t)$ is a scalar function, $\bar{\mathbf{h}}$ and $\bar{\boldsymbol{\beta}}$ are vectors, and matrices $\bar{\mathbf{K}}$ and $\bar{\boldsymbol{\Sigma}}$ need not be diagonal.

Then, the following affine transformation $\mathrm{T}(\boldsymbol{\zeta}_t) = \mathbf{L}\boldsymbol{\zeta}_t + \boldsymbol{\varphi}(t)$ may be applied to the original state vector $\boldsymbol{\zeta}_t$ to obtain a new state vector $\mathbf{x}_t = \mathrm{T}(\boldsymbol{\zeta}_t)$. If the matrix L is invertible, then there exists a one-to-one correspondence between the state variables of the two models. The new model is

$$\log S_t = \mathbf{h}'\mathbf{x}_t + \mu(t) \tag{6}$$

$$d\mathbf{x}_t = (-\mathbf{K}\mathbf{x}_t + \boldsymbol{\beta})dt + \boldsymbol{\Sigma}\, d\mathbf{w}_t \tag{7}$$

[2]Obviously, this assumption could easily be relaxed in order to study stationary models of commodity prices. Actually, when the model was estimated with this assumption relaxed, no significant differences were found. Also, the model performs well for other commodities, as will be shown later when calibrated with copper futures.

[3]It is assumed for simplicity that the risk premium is constant. However, this could be extended to any linear function of the state variables to reflect a possible correlation between spot prices, convenience yields, and interest rates (Casassus & Collin-Dufresne, 2005).

where $\mathbf{h}' = \bar{\mathbf{h}}'\mathbf{L}^{-1}, \mu(t) = \bar{\mu}(t) - \mathbf{h}'\boldsymbol{\varphi}(t), \mathbf{K} = \mathbf{L}\bar{\mathbf{K}}\mathbf{L}^{-1}, \boldsymbol{\beta} = \mathbf{K}\boldsymbol{\varphi}(t) + \mathbf{L}\bar{\boldsymbol{\beta}}$, $d\mathbf{w}_t\, d\mathbf{w}_t' = \boldsymbol{\Omega}\, dt, \boldsymbol{\Sigma\Omega\Sigma} = \mathbf{L}\bar{\boldsymbol{\Sigma}}\,\bar{\boldsymbol{\Omega}}\,\bar{\boldsymbol{\Sigma}}\mathbf{L}'$ and $\boldsymbol{\Sigma} = \mathbf{L}\bar{\boldsymbol{\Sigma}}$.

From the analysis of Dai and Singleton (2000) it follows that there exists an affine transformation $\mathbf{T}(\cdot)$ that rewrites any model to another one with the maximum number of parameters that can be econometrically identified. In particular, if the reversion matrix $\mathbf{K}$ is restricted to have termwise different eigenvalues, then an arbitrary N-factor model can always be transformed to a model like the one presented in Equations (1) and (2).[4]

For example, consider the Gibson and Schwartz (1990) model:

$$\log S_t = \mathbf{h}'\boldsymbol{\zeta}_t \tag{8}$$

$$d\boldsymbol{\zeta}_t = (-\bar{\mathbf{K}}\boldsymbol{\zeta}_t + \boldsymbol{\beta})dt + \bar{\boldsymbol{\Sigma}}d\bar{\mathbf{w}}_t \tag{9}$$

where $\mathbf{h}' = (1 \quad 0)$, $\bar{\mathbf{K}} = \begin{pmatrix} 0 & 0 \\ 0 & \kappa \end{pmatrix}$, $\boldsymbol{\beta} = \begin{pmatrix} \mu - 1/2\sigma_1^2 \\ \kappa\alpha \end{pmatrix}$, $\bar{\boldsymbol{\Sigma}} = \begin{pmatrix} \bar{\sigma}_1 & 0 \\ 0 & \bar{\sigma}_2 \end{pmatrix}$ and $d\bar{\mathbf{w}}_t d\bar{\mathbf{w}}_t' = \bar{\boldsymbol{\Omega}}dt = \begin{pmatrix} 1 & \bar{\rho} \\ \bar{\rho} & 1 \end{pmatrix} dt$.

To obtain the relationship between the two models, the following affine transformation is applied over the original state variable vector $\boldsymbol{\zeta}_t$:

$$\mathbf{x}_t = \mathbf{L}\boldsymbol{\zeta}_t + \boldsymbol{\varphi} \tag{10}$$

where $\mathbf{L} = \begin{pmatrix} 1 & -1/\kappa \\ 0 & 1/\kappa \end{pmatrix}$ and $\boldsymbol{\varphi} = \begin{pmatrix} (\mu - 1/2\sigma_1^2 - \alpha)t - \alpha/\kappa \\ \alpha/\kappa \end{pmatrix}$.

The new vector $\mathbf{x}_t$ corresponds to the state variables in the new model. This transformation is invertible, and therefore it establishes a one-to-one correspondence between the state variables of the two models, obtaining

$$\log S_t = \mathbf{1}'\mathbf{x}_t + \left(\mu - \frac{1}{2}\sigma_1^2 - \alpha\right)t \tag{11}$$

$$d\mathbf{x}_t = -\mathbf{K}\mathbf{x}_t dt + \boldsymbol{\Sigma}\, d\mathbf{w}_t \tag{12}$$

where $\mathbf{K} = \begin{pmatrix} 0 & 0 \\ 0 & \kappa \end{pmatrix}$, $\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix}$ and $d\mathbf{w}_t\, d\mathbf{w}_t' = \boldsymbol{\Omega}\, dt = \begin{pmatrix} 1 & \rho_{21} \\ \rho_{21} & 1 \end{pmatrix} dt$.

[4]The reversion matrix is assumed to have termwise different eigenvalues to obtain simpler valuation formulas for futures contracts.

The existing relationship between the variance–covariance parameters of the two models is given by $\Sigma\Omega\Sigma = \mathbf{L}\overline{\Sigma}\,\overline{\Omega}\,\overline{\Sigma}\,\mathbf{L}'$. Assuming a constant risk premium λ_i for each state variable, the risk-adjusted process of the Gibson and Schwartz (1990) model is

$$d\mathbf{x}_t = (-\overline{\mathbf{K}}\boldsymbol{\zeta}_t + \boldsymbol{\beta} - \overline{\boldsymbol{\lambda}})dt + \overline{\Sigma}\,d\overline{\mathbf{w}}_t \tag{13}$$

Under the equivalent martingale measure the drift of the spot price of a commodity must be equal to the difference between the instantaneous interest rate r, which is assumed constant by Gibson and Schwartz (1990), and its instantaneous convenience yield, so the following relation must hold:

$$\mu - \lambda = r \tag{14}$$

The risk-adjusted process of the new model is:

$$d\mathbf{x}_t = (-\mathbf{K}\mathbf{x}_t - \boldsymbol{\lambda})dt + \Sigma\,d\mathbf{w}_t \tag{15}$$

Then, it can be shown that the relationship between the risk premiums of the two models is $\boldsymbol{\lambda} = \mathbf{L}\overline{\boldsymbol{\lambda}}$.

Under this new representation, it follows from Equations (11) and (14) that it is not possible to estimate μ, α, and λ independently. One of the three parameters, or a linear combination of them, must be exogenously specified. In the article, Gibson and Schwartz (1990) estimate from government bond data the instantaneous risk-free interest rate $r = \mu - \lambda$.

Futures Prices

The price of a futures contract at time t and maturing at T can then be found as the expected value of the spot price under the risk-neutral measure (Cox, Ingersoll, & Ross, 1981):

$$F(\mathbf{x}_t, t, T) = \mathrm{E}_t^Q(S_T) \tag{16}$$

As shown in the Appendix, the expected value in Equation (16) can be computed as

$$F(\mathbf{x}_t, t, T) = \exp\Bigg(x_1(t) + \sum_{i=2}^{N} e^{-\kappa_i(T-t)}x_i(t) + \mu t + \left(\mu - \lambda_1 + \frac{1}{2}\sigma_1^2\right)(T-t) - \sum_{i=2}^{N}\frac{1 - e^{-\kappa_i(T-t)}}{\kappa_i}\lambda_i + \frac{1}{2}\sum_{i \cdot j \neq 1}\sigma_i\sigma_j\rho_{ij}\frac{1 - e^{-(\kappa_i+\kappa_j)(T-t)}}{\kappa_i + \kappa_j}\Bigg) \tag{17}$$

One important advantage of this model is its tractability, with explicit futures price formulas even for an arbitrary number of factors. In addition, the logarithm of the futures price is a linear function of state variables, which is useful when estimating the model with a Kalman-filter–based procedure. Because the state variables have a multivariate normal distribution, any linear combination of state variables will also distribute normal, allowing maximum-likelihood techniques to be used.

Finally, the model volatility term structure of futures returns can be obtained from Equations (2) and (17):

$$\sigma_F^2(\tau) = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_i \sigma_j \rho_{ij} e^{-(\kappa_i+\kappa_j)\tau} \tag{18}$$

Given that $\kappa_1 = 0$, as the maturity of a futures contract grows, the volatility of futures returns converges to a constant given by σ_1, which is the volatility of the first state variable.

ESTIMATION METHODOLOGY

The Kalman filter is an estimation methodology that recursively calculates optimal estimates of unobservable state variables with the use of all past information. Consistent parameter estimates can be obtained by maximizing the likelihood function of error innovations. In the finance literature, the Kalman filter has been used to estimate and implement stochastic models of commodities (Schwartz, 1997; Schwartz & Smith, 2000; Sorensen, 2002), interest rates (Babbs & Nowman, 1999; Cortazar, Schwartz, & Naranjo, 2003; de Jong, 2000; de Jong & Santa-Clara, 1999; Duan & Simonato, 1999; Geyer & Pichler, 1999; Lund, 1994, 1997), and other relevant economic variables (Pennacchi, 1991). Although widely used in a complete panel-data setting, most literature has not focused on using the Kalman filter where there are missing observations in the panel, a common feature in many commodity futures markets.

One of the characteristics of the Kalman filter is that state variables estimates are obtained with the use of a rich information set that includes past information and not only current prices. Moreover, it can allow for measurement errors in observable variables that may be induced by market imperfections or by the inability of a model with a restricted number of factors to explain the whole structure of contemporaneous observations.

The Kalman filter may be applied to dynamic models that are in a state-space representation. The measurement equation relates a vector of observable variables z_t with a vector of state variables $\mathbf{x}_t$:

$$z_t = \mathbf{H}_t \mathbf{x}_t + \mathbf{d}_t + \mathbf{v}_t \qquad \mathbf{v}_t \sim N(\mathbf{0}, \mathbf{R}_t) \tag{19}$$

where z_t is a $m_t \times 1$ vector, H_t is a $m_t \times n$ matrix, x_t is a $n \times 1$ vector, d_t is a $m_t \times 1$ vector, v_t is a $m_t \times 1$ vector of serially uncorrelated Gaussian disturbances with mean 0 and covariance matrix R_t of dimension $m_t \times m_t$, and m_t is the number of observations available at time t.

Measurement Equation (19) assumes the existence of a linear relation between observed variables and state variables. As noted above, in this model the logarithm of futures prices is a linear function of state variables. Nevertheless, the Kalman filter could be modified (Harvey, 1989) to allow for nonlinear measurement equations, as would be the case if, for example, commodity option prices alone, or in combination with futures prices, were used as observations.

The transition equation describes the stochastic process followed by the state variables:

$$x_t = A_t x_{t-1} + c_t + \varepsilon_t \qquad \varepsilon_t \sim N(0, Q_t) \tag{20}$$

where A_t is an $n \times n$ matrix, c_t is an $n \times 1$ vector, and ε_t is an $n \times 1$ vector of serially uncorrelated Gaussian disturbances with mean 0 and covariance matrix. Given this state-space representation, the Kalman filter calculates optimal estimates $\hat{x}_t$ of state variables and the variance-covariance matrix $P_t = E(x_t - \hat{x}_t)(x_t - \hat{x}_t)'$.

The Kalman filter then works recursively, using the previous estimations. First, the one-step-ahead prediction at time t of the state variables $\hat{x}_{t|t-1}$ and its error variance–covariance matrix $P_{t|t-1}$ given all information up to time $t - 1$ are computed:

$$\hat{x}_{t|t-1} = A_t \hat{x}_{t-1} + c_t \tag{21}$$

$$P_{t|t-1} = A_t P_{t-1} A'_t + Q_t \tag{22}$$

This allows for the calculation of one-step-ahead prediction of observed variables:

$$\hat{z}_{t|t-1} = H_t \hat{x}_{t|t-1} + d_t \tag{23}$$

These calculations only consider the dynamic properties of state variables and are not affected by the dimension of the vector of observable variables. The prediction error or innovation ν_t and its associated variance–covariance matrix F_t are:

$$\nu_t = z_t - \hat{z}_{t|t-1} \tag{24}$$

$$F_t = H_t P_{t|t-1} H'_t + R_t \tag{25}$$

When futures contracts are used as observations, the measurement equation gives the futures price as a function of state variables and its maturity, allowing all observed prices to be used in the estimation process. With the use of Equation (17), the m_t row vectors of the matrix $\mathbf{H}_t$ and the elements of the vector $\mathbf{d}_t$ can be computed.

In addition, the covariance matrix of measurement error $\mathbf{R}_t$ must be parametrized. For this purpose, the Babbs and Nowman (1999) approach is followed, where it is assumed that all measurement errors are independent and have the same variance, ξ^2, inducing a diagonal covariance matrix $\mathbf{R}_t$. Although this assumption could be relaxed, there is a tradeoff between the accuracy and the complexity of the model in terms of the number of parameters that need to be estimated.

The optimal estimates are then computed in what is called the update step:

$$\hat{\mathbf{x}}_t = \hat{\mathbf{x}}_{t|t-1} + \mathbf{P}_{t|t-1}\mathbf{H}_t'\mathbf{F}_t^{-1}\boldsymbol{\nu}_t \tag{26}$$

$$\mathbf{P}_t = \mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\mathbf{H}_t'\mathbf{F}_t^{-1}\mathbf{H}_t\mathbf{P}_{t|t-1} \tag{27}$$

Note that the above calculations can be performed even if the number of observations varies with time and that the accuracy of the estimation, measured by the variance of the estimation error, increases with the number of observations that are available to update the filter.

The estimation of model parameters $\hat{\boldsymbol{\Psi}}$ is obtained by maximizing the log-likelihood function of innovations:

$$\log L(\boldsymbol{\Psi}) = -\frac{1}{2}\sum_t \log|\mathbf{F}_t| - \frac{1}{2}\sum_t \boldsymbol{\nu}_t'\mathbf{F}_t^{-1}\boldsymbol{\nu}_t \tag{28}$$

where $\boldsymbol{\Psi}$ represents a vector containing unknown parameters.

EMPIRICAL RESULTS

Data

The data used in this study consist of all daily light sweet crude oil futures prices traded at NYMEX from January 1992 to December 2004. There are currently 35 contracts traded for different maturities ranging from 1 to 30 months, and 3, 4, 5, 6, and 7 years. However, from 1992 to 1996, the maximum maturity traded at NYMEX was only 4 years. In 1997, new contracts were introduced to include maturities up to 7 years.

TABLE I
Average Number of Daily Observations and Maximum Maturity Available of Light Sweet Crude Oil Futures Contracts for Panel A (1992–2001), B (1992–1996), C (1997–2001), and D (2002–2004)

	Year	Average number of daily observations	Maximum maturity (years)
Panel A			
Panel B	1992	22	3
	1993	22	3
	1994	21	3
	1995	25	4
	1996	31	4
Panel C	1997	34	7
	1998	31	7
	1999	31	7
	2000	33	7
	2001	34	7
Panel D	2002	34	7
	2003	34	7
	2004	33	7

The data are divided into four different panels in Table I. Panel A includes all futures contracts traded between 1992 and 2001, which correspond to 70,584 observations. In order to analyze the effect (if any) in parameter estimates of the introduction of long-term contracts in 1997, the data are divided into Panel B (1992–1996) and Panel C (1997–2001), with 30,424 and 40,160 observations, respectively.[5] Panel D is used for out-of-sample testing purposes only, and includes futures traded between 2002 and 2004, with 24,947 observations. Table I presents a description of the data showing the average number of daily observations and the maximum maturity available for each year during the 1992–2004 period.

The time series of the spot oil price from 1992 to 2004 is displayed in Figure 1, where it is possible to appreciate the high volatility exhibited by oil. Spot prices reach a maximum of \$55 per barrel, and the minimum is as low as \$10 per barrel. The spot price, defined as the value of an expiring futures contract, is usually unobservable and must be estimated. The simplest way of doing it is to use the closest-to-maturity futures contract as a proxy for the oil spot price. A more rigorous way to estimate the spot price is to set $T = t$ in Equation (17). The price of the closest-to-maturity contract and the theoretical spot price derived from the four-factor model are similar but not identical, as shown in Figure 1.

[5]The data could also be divided with alternative criteria, like structural breaks in oil prices (Serletis, 1992).

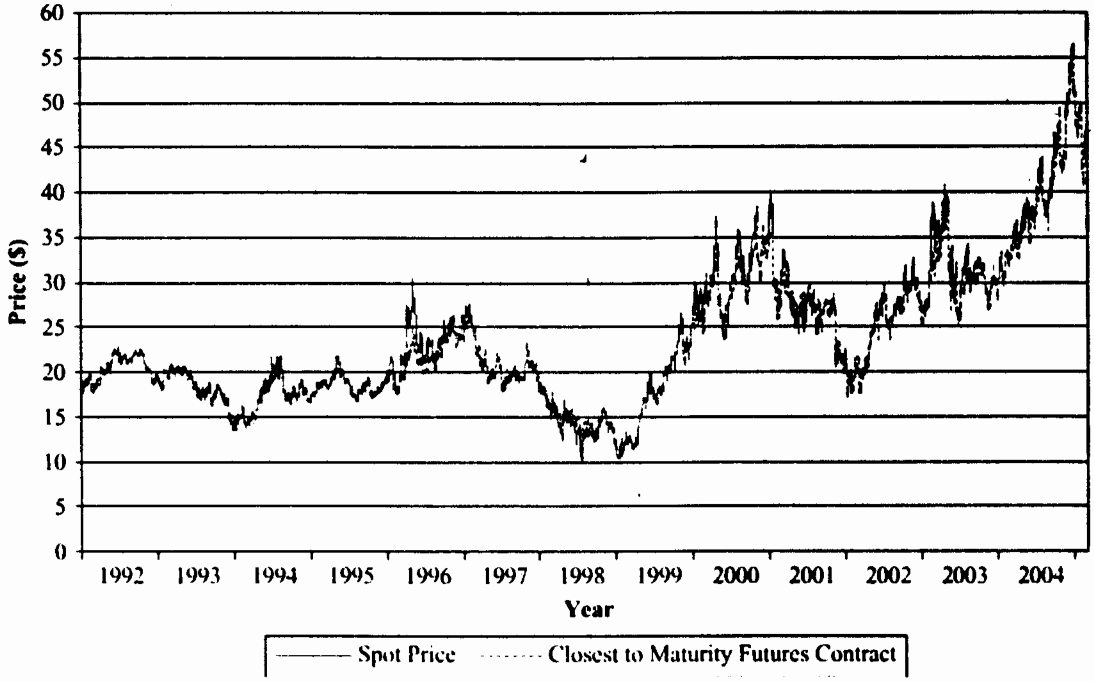

FIGURE 1
Time series of the oil spot price. The figure displays the historic evolution of the theoretical oil spot price, calculated from the four-factor model with the use of parameter estimates from Panel A, and compares it with the closest-to-maturity futures contract from January 1992 to December 2004. The spot price, although very similar, is not identical to the closest-to-maturity futures contract.

Parameter Estimation

The model is estimated in Table II with the use of one, two, three, and four factors, for three different panels: Panel A from 1992 to 2001, Panel B from 1992 to 1996, and Panel C from 1997 to 2001. The remaining data are used later for robustness tests of the model. The Kalman filter described earlier is used to estimate unobserved state variables, and parameters are obtained by maximizing the likelihood function of futures price innovations.

Parameter estimates are shown in Table II with standard errors in parentheses. From this table it can be seen that most parameters are stable across different panels, which shows the reliability of the model when applied to the oil market. All mean reversion parameters, κ_i, for all three panels, are highly significant and show the existence of strong mean reversion in oil prices. Volatility parameters σ_i are also highly significant and stable across panels. Correlation parameters ρ_{ij} are almost all significant. As found in the literature (Schwartz, 1997), the long-term growth rate parameter μ and most risk premium parameters λ_i are not

TABLE II

Parameter Estimates From Oil Futures Data for Different Panels

	Panel A				Panel B				Panel C			
	Four Factors	*Three Factors*	*Two Factors*	*One Factor*	*Four Factors*	*Three Factors*	*Two Factors*	*One Factor*	*Four Factors*	*Three Factors*	*Two Factors*	*One Factor*
κ_2	0.415 (0.002)	0.485 (0.002)	0.680 (0.002)	—	0.681 (0.013)	1.041 (0.004)	1.258 (0.013)	—	0.415 (0.003)	0.451 (0.002)	0.661 (0.002)	—
κ_3	1.201 (0.005)	1.636 (0.006)	—	—	1.283 (0.012)	3.776 (0.013)	—	—	1.239 (0.010)	1.448 (0.006)	—	—
κ_4	5.471 (0.039)	—	—	—	7.255 (0.038)	—	—	—	3.545 (0.073)	—	—	—
σ_1	0.191 (0.003)	0.192 (0.004)	0.162 (0.003)	0.136 (0.004)	0.150 (0.003)	0.147 (0.003)	0.127 (0.003)	0.111 (0.004)	0.210 (0.004)	0.206 (0.005)	0.195 (0.005)	0.159 (0.007)
σ_2	0.207 (0.004)	0.175 (0.004)	0.191 (0.003)	—	0.230 (0.009)	0.156 (0.004)	0.179 (0.004)	—	0.253 (0.007)	0.211 (0.008)	0.337 (0.006)	—
σ_3	0.305 (0.007)	0.507 (0.008)	—	—	0.246 (0.010)	0.160 (0.004)	—	—	0.334 (0.009)	0.705 (0.011)	—	—
σ_4	0.260 (0.005)	—	—	—	0.214 (0.005)	—	—	—	0.355 (0.009)	—	—	—
ρ_{21}	−0.336 (0.025)	−0.323 (0.025)	−0.045 (0.024)	—	−0.258 (0.032)	0.027 (0.026)	0.287 (0.014)	—	−0.279 (0.025)	−0.192 (0.040)	−0.188 (0.037)	—
ρ_{31}	0.138 (0.029)	0.310 (0.051)	—	—	0.242 (0.033)	0.186 (0.014)	—	—	0.070 (0.026)	0.031 (0.056)	—	—
ρ_{32}	−0.423 (0.025)	−0.068 (0.051)	—	—	−0.714 (0.025)	−0.006 (0.020)	—	—	−0.442 (0.027)	−0.217 (0.085)	—	—
ρ_{41}	−0.010 (0.027)	—	—	—	0.036 (0.035)	—	—	—	0.018 (0.025)	—	—	—
ρ_{42}	0.420 (0.023)	—	—	—	0.348 (0.035)	—	—	—	0.572 (0.028)	—	—	—
ρ_{43}	−0.338 (0.027)	—	—	—	−0.162 (0.040)	—	—	—	−0.554 (0.021)	—	—	—
μ	0.004 (0.059)	0.006 (0.060)	0.006 (0.051)	0.010 (0.043)	−0.007 (0.066)	−0.013 (0.065)	−0.016 (0.065)	0.013 (0.050)	0.020 (0.093)	0.027 (0.092)	0.026 (0.087)	0.004 (0.072)
λ_1	0.013 (0.059)	0.015 (0.060)	0.012 (0.051)	0.036 (0.043)	−0.019 (0.066)	−0.027 (0.065)	−0.033 (0.065)	0.025 (0.050)	0.035 (0.093)	0.041 (0.092)	0.039 (0.087)	0.035 (0.072)
λ_2	0.002 (0.054)	0.015 (0.047)	0.056 (0.053)	—	−0.012 (0.081)	0.129 (0.059)	0.176 (0.068)	—	0.026 (0.080)	0.039 (0.069)	0.129 (0.120)	—
λ_3	0.117 (0.089)	0.168 (0.152)	—	—	0.169 (0.095)	0.085 (0.068)	—	—	0.185 (0.128)	0.192 (0.280)	—	—
λ_4	−0.073 (0.079)	—	—	—	0.101 (0.091)	—	—	—	−0.206 (0.143)	—	—	—
ϵ	0.003 (0.000)	0.005 (0.000)	0.015 (0.000)	0.060 (0.000)	0.002 (0.000)	0.003 (0.000)	0.009 (0.000)	0.037 (0.000)	0.004 (0.000)	0.006 (0.000)	0.015 (0.000)	0.072 (0.000)
log L	357,087	323,250	256,237	162,645	170,067	153,226	125,323	84,155	198,754	182,156	145,438	85,204

Note. Standard errors are shown in parentheses.

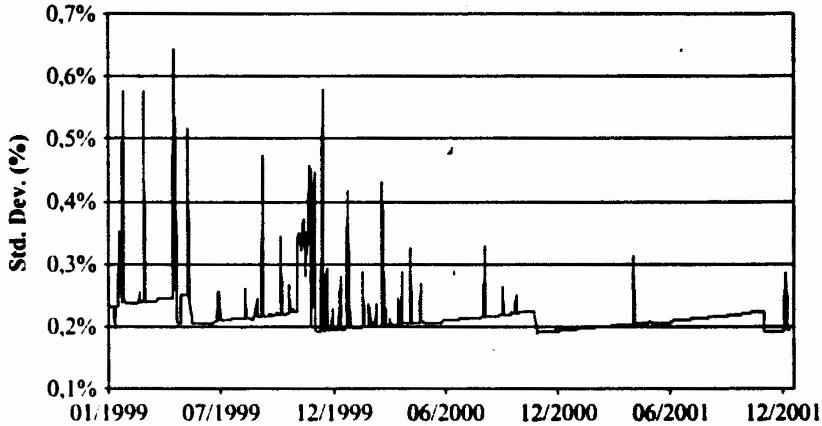

FIGURE 2
Time series of the standard deviation for state variable x_1. The figure displays the time evolution from January 1999 to December 2001 of the standard deviation for state variable x_1 in the four-factor model, which is obtained from the first entry of matrix $\mathbf{P}_t$, the covariance matrix of state variable estimation errors.

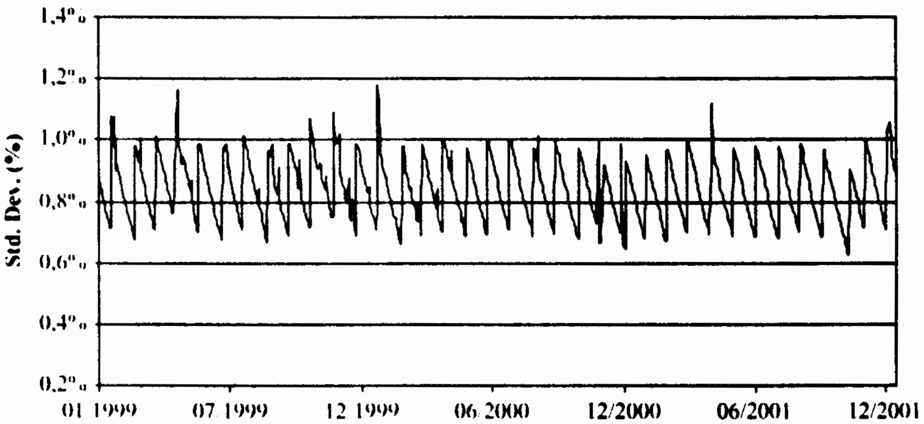

FIGURE 3
Time series of the standard deviation for state variable x_4. The figure displays the time evolution from January 1999 to December 2001 of the standard deviation for state variable x_4 in the four-factor model, which is obtained from the last diagonal entry of matrix $\mathbf{P}_t$, the covariance matrix of state variable estimation errors.

statistically significant. The standard deviation measurement error parameter ξ is small, although very significant.[6]

Figures 2 and 3 display the standard deviation of the estimation error, calculated as the squared root of the diagonal elements of matrix $\mathbf{P}_t$, for two different state variables in the four-factor model. This is important when incomplete data sets are used. As expected, a more complete data set induces a lower estimation error, but the estimation error of a particular

[6]In this article, this parameter corresponds to the root-mean-squared error (RMSE) for the whole sample.

state variable should depend on the availability of futures contracts for the specific maturity range represented by that particular state variable.

Figure 2 plots from January 1999 to December 2001 the estimation error of x_1, which heavily depends on the availability of long-term futures contracts. It can be seen that the estimation error sharply increases for some dates corresponding to days when long-term contracts are not traded. In addition, it shows that the standard deviation increases between two consecutive issues of long-term contracts, because a shorter remaining maturity provides less information of long-term behavior.

Similarly, Figure 3 displays the time evolution from January 1999 to December 2001 of the estimation error for state variable x_4, which has the highest mean reversion among all state variables. Because short-term futures contracts usually trade every day, there are no abnormal spikes like the ones observed in Figure 2. This standard deviation presents a decreasing seasonal pattern with a 1-month cycle following monthly emissions of short-term futures contracts.

Model Robustness

Besides the stability of parameter estimates, the performance of this model and estimation procedure is measured by analyzing the fit to the observed futures prices term structure and the empirical volatility term structure of futures returns.

Figures 4 and 5 show the fit of the models for two dates. These specific dates were chosen as examples of market conditions when oil

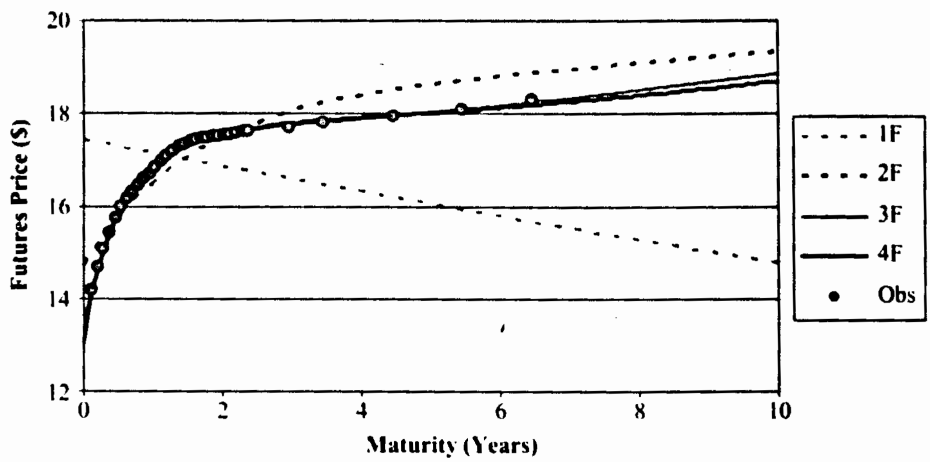

FIGURE 4
Estimated and observed oil futures prices on 06/30/1998. The figure displays the theoretical term structure of futures prices on 06/30/1998 with the use of one-, two-, three-, and four-factor models, and compares it to observed futures prices when the term structure is in contango.

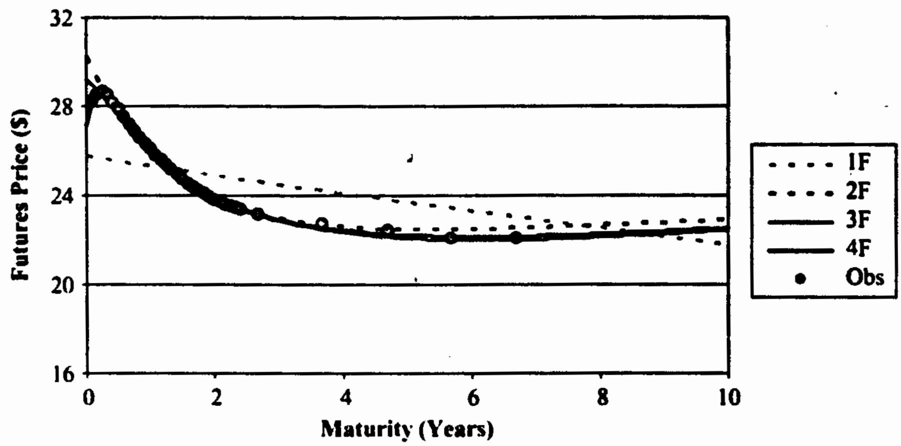

FIGURE 5

Estimated and observed oil futures prices on 04/11/2001. The figure displays the theoretical term structure of futures prices on 04/11/2001 with the use of one-, two-, three-, and four-factor models, and compares it to observed futures prices when the term structure is in backwardation.

TABLE III

In-Sample RMSE and Bias for Panels A (1992–2001), B (1992–1996), and C (1997–2001)

	RMSE	*Bias*
Panel A		
1F	5.92%	0.0004%
2F	1.46%	−0.0004%
3F	0.51%	−0.0002%
4F	0.29%	−0.0001%
Panel B		
1F	3.70%	−0.0009%
2F	0.87%	0.0000%
3F	0.31%	0.0000%
4F	0.16%	0.0000%
Panel C		
1F	7.12%	0.0010%
2F	1.46%	−0.0003%
3F	0.53%	−0.0003%
4F	0.35%	−0.0002%

futures term structures exhibited strong contango or backwardation. It can be seen that one- and two-factor models cannot fit observed futures prices accurately on these dates, whereas three- and four-factor models fit them very well.

Table III shows the root-mean-squared error (RMSE) and bias of model futures estimates for in-sample data. It can be seen that futures

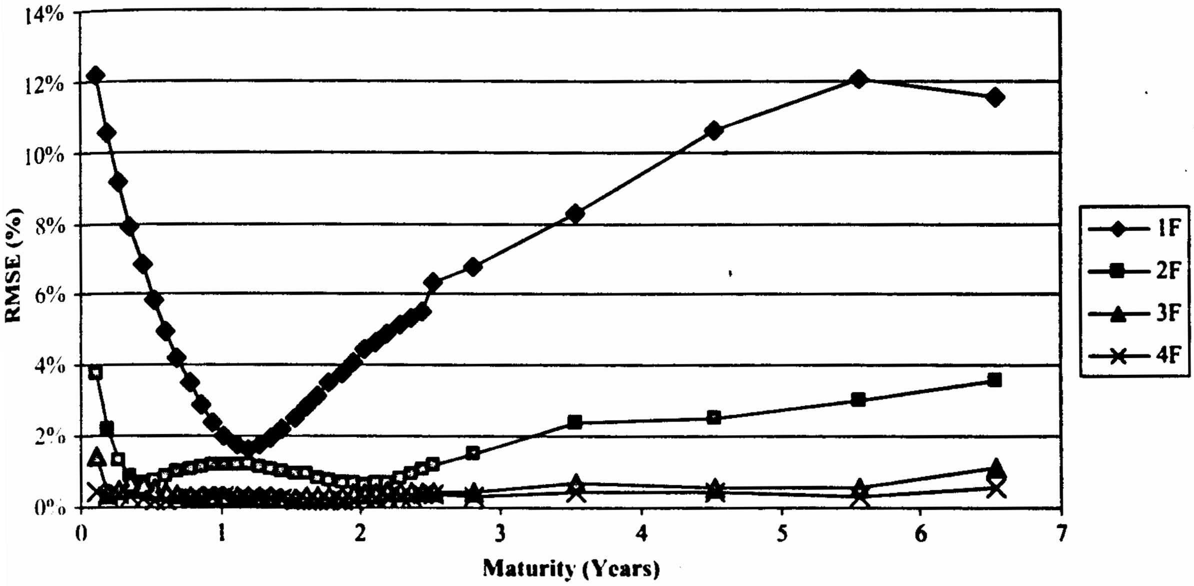

FIGURE 6
Root-mean-squared errors for each model. The figure is obtained by calculating the in-sample root-mean-squared errors (RMSE) by maturity for the 35 existing oil futures contracts and for each model with the use of Panel A (1992–2001).

price estimations are unbiased for all models and panels and exhibit a RMSE of less than 1% for three- and four-factor models. For example, in Panel A, considering an average spot oil price of $21, the RMSE corresponds to an error of only $0.07 for the four-factor model. On the other hand, the one-factor model exhibits a high RMSE ranging from 3.70 to 7.12%. Figure 6 displays the RMSE by maturity for each of the models. It can seen that the inclusion of more factors improves the fit for all maturities, especially for short- and long-term contracts.

Another way to measure model stability is to compare the RMSE of out-of-sample and in-sample estimations. For this purpose the RMSE for years 2002, 2003, and 2004 is calculated with the use of Panel A parameters. Table IV displays these results and shows that the RMSE for these years is similar to the RMSE obtained for 2001.

It is important to note that these results are obtained under the assumption that the spot price is nonstationary, which seems reasonable for oil (Schwarz & Szakmary, 1994). Moreover, Bessembinder et al. (1995) find large and significant mean reversion in agricultural commodities and crude oil futures markets, smaller but statistically significant mean reversion in metals futures, and weak mean reversion in financial futures markets. Thus, one may be concerned that the nonstationarity of the model combined with the embedded assumption of mean

TABLE IV

In-Sample RMSE for the Year 2001 and Out-of-Sample RMSE for Years 2002–2004 Calculated with Panel A (1992–2001) Parameters for Different Number of Factors

	2001 (In sample)	*2002*	*2003*	*2004*
1F	5.43%	4.38%	5.93%	5.52%
2F	1.38%	1.22%	1.88%	1.35%
3F	0.60%	0.66%	0.71%	0.54%
4F	0.36%	0.53%	0.46%	0.37%

TABLE V

In-Sample RMSE for the Year 2001 and Out-of-Sample RMSE for Years 2002 to 2004 for Copper Futures With the Use of Different Numbers of Factors

	2001 (In sample)	*2002*	*2003*	*2004*
1F	2.46%	2.36%	1.37%	5.88%
2F	0.25%	0.12%	0.17%	1.29%
3F	0.16%	0.07%	0.08%	0.44%

reversion may make the model fit the oil futures well, but not other commodity futures, like metals.[7]

To address this issue, Table V presents results for the same model estimated with the use of copper futures for 1, 2, and 3 factors. In particular, the data used in this estimation consist of daily copper futures traded at NYMEX from January 1992 to December 2001. The table displays out-of-sample RMSE for the years 2002, 2003, and 2004, and also in-sample RMSE for the year 2001. It can be seen that results similar to those obtained for oil are found for copper, suggesting that the model behaves well for some metals. This is also consistent with the results reported by Schwartz (1997).

As a last measure of robustness, the model volatility term structure of futures returns is calculated from Equation (18), and compared to the empirical volatilities $\hat{\sigma}_F^2(\tau)$ obtained directly from observed futures prices:

$$\hat{\sigma}_F^2(\tau) = \frac{1}{\Delta t} \sum_{i=1}^{N} (\log(F(t_i, \tau)/F(t_i - \Delta t, \tau)) - \overline{\mu})^2 \qquad (29)$$

Figure 7 shows, for each model and panel, the theoretical and empirical volatility term structures. It can be seen that one- and two-factor models do not fit the empirical volatility term structure well. Although the volatility term structure in Panel B (1992–1996) fits the

[7] The authors thank the referee for this comment.

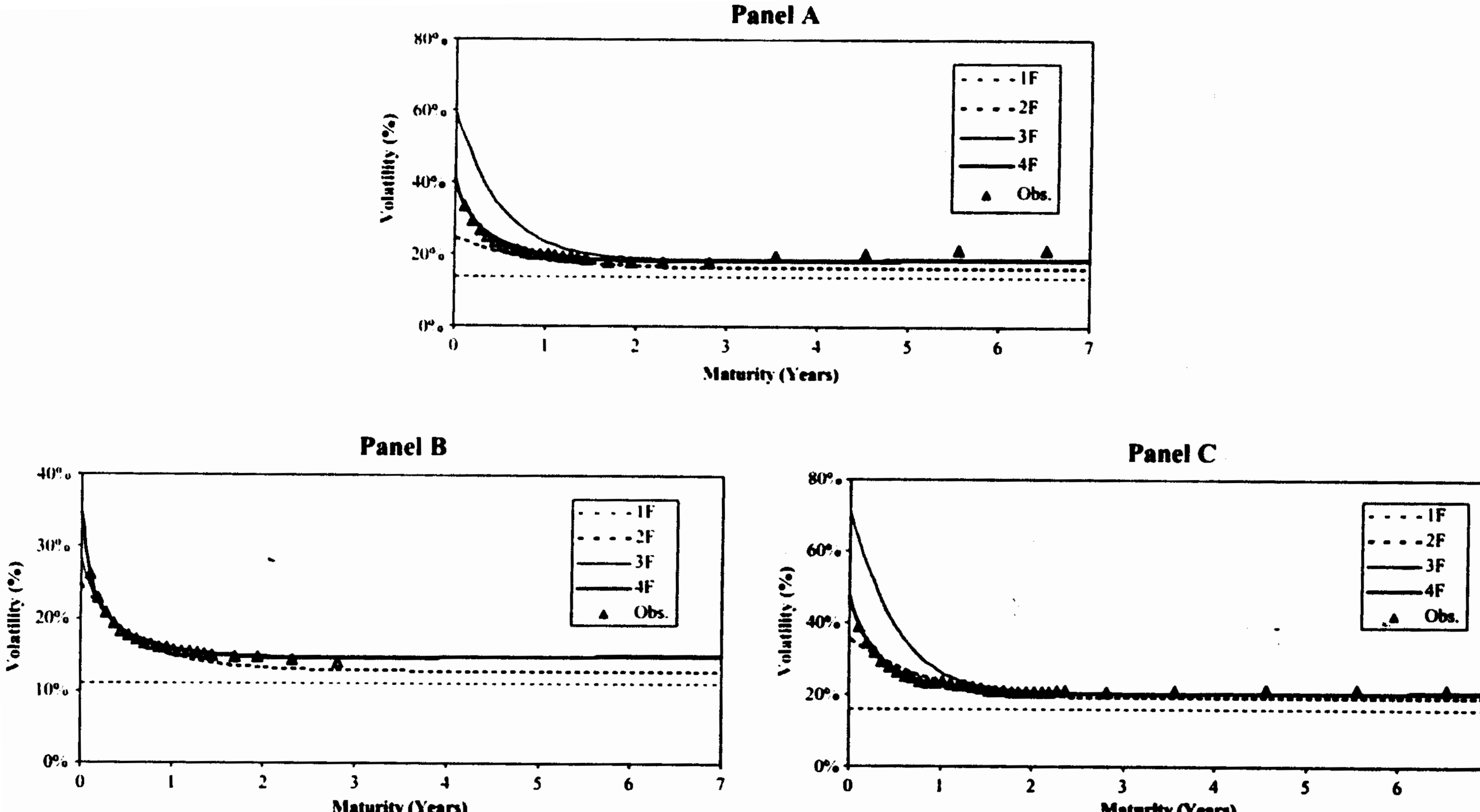

FIGURE 7
Volatility term structure of oil futures returns. The figures presents the theoretical volatility term structure of futures returns for each model, and the empirical volatility of futures returns for each contract in Panel A (1992–2001), Panel B (1992–1996), and Panel C (1997–2001).

three-factor model very well, it does not perform well in Panel C (1997–2001), as it overestimates the short-term volatility. This translates into an overestimated short-term volatility for Panel A, as it is the union of Panels B and C. On the contrary, the four-factor model closely fits the empirical volatility term structure across all panels.

A possible explanation for the failure of the three-factor model to fit the volatility term structure might be a structural break in the volatility of oil futures returns during the 1992–2001 period.[8] One way to test this hypothesis is to compare model estimates between the two different periods 1992–1996 and 1997–2001. This could easily be done by comparing the likelihood functions, and seeing whether this difference is significant. One may be concerned, however, that because the current estimation procedure uses a very large number of data points, it will reject any hypothesis of no structural change. On the other hand, it may be interesting to use as much information as possible from the whole futures term structure in testing for structural change.

The structural break is tested by comparing the time-series covariance matrix of state variables for each sample, and then computing a Wald test statistic. In particular, the Wald statistic is computed with the use of the GMM procedure applied to the time-series dynamics of each state variable, and carried out for each model. This generates Wald-statistic values of 456.05, 171.70, 115.19, and 97.51 for the four-, three-, two-, and one-factor models, respectively. Given that critical values at 95% for this test are 18.31, 12.59, 7.81, and 3.84, respectively, the null hypothesis of no structural change is clearly rejected.

In addition, the presence of volatility parameters in the *valuation formula* of futures prices might also cause empirical and theoretical volatility term structures to diverge. Because the estimation is performed by improving the likelihood function of price innovations, there is always a trade-off between improving the pricing of futures contracts and the time-series volatility estimation. A larger number of factors gives more flexibility to adjust first and second moments simultaneously, hence explaining why the four-factor model outperforms the three-factor one in fitting the volatility term structure.

CONCLUSIONS

This article studies the ability of an N-factor Gaussian model to explain the stochastic behavior of oil futures prices when estimated with the use of all available price information. In recent years, oil futures markets

[8]The authors thank the referee for pointing this out.

have included new futures contracts with longer maturities that do not have historical prices. In addition, not all futures contracts trade every day. To include all data without discarding or aggregating prices, a Kalman filter estimation procedure that allows for a time-dependent number of observations is used.

The model is calibrated with the use of all daily light sweet crude oil futures prices traded at NYMEX during the 10-year period from January 1992 to December 2001. The model is estimated with the use of one, two, three and four factors, for the three different panels: from 1992 to 2001, from 1992 to 1996, and from 1997 to 2001. It is found that most parameter estimates are significant and stable across different panels, as opposed to the long-term growth rate and most risk premium parameters, which are not. Moreover, out-of-sample errors for the 2002–2004 period are similar to in-sample errors, supporting the stability of the model. In addition, the model also performs well for copper futures.

Empirical results show that one and two-factor models fail to accurately fit observed prices and the volatility term structure. The three-factor model, while explaining market prices very well, overestimates the short-term volatility in some panels, which may be attributed to a structural change in oil prices occurring after 1997. Finally, the four-factor model performs well, explaining the stochastic behavior of oil prices.

In general, the model works well in estimating the term structure of oil futures prices and the volatility term structure of oil futures returns. As such, the model could be useful for oil producers and consumers, and also financial intermediaries like futures traders, in valuing and hedging oil contingent claims. For example, the model could be used to value oil-linked financial contracts with option-like characteristics, or to implement long-term hedging strategies with existing futures contracts.

APPENDIX A

This Appendix deduces Equation (17) with the use of Equation (16). Because the conditional distribution for the spot price S_T is lognormal, it follows that

$$E_t^Q(S_T) = \exp\left(E_t^Q(Y_T) + \frac{1}{2}V_t^Q(Y_T)\right) \tag{30}$$

where $Y_T = \log(S_T)$, $E_t(Y_T) = (1')(E_t^Q(x_T)) + \mu T$ and $\mathrm{Var}_t(Y_T) = (1') \times (\mathrm{Cov}_t^Q(x_T))(1)$.

From Equation (3) the conditional moments of $\mathbf{x}_t$ are

$$\mathbf{E}_t(\mathbf{x}_T) = \mathrm{e}^{-\mathbf{K}(T-t)}\mathbf{x}_t - \left(\int_0^{T-t} \mathrm{e}^{-\mathbf{K}\tau} d\tau\right)\boldsymbol{\lambda} \tag{31}$$

$$\mathbf{Cov}_t(\mathbf{x}_T) = \int_0^{T-t} \mathrm{e}^{-\mathbf{K}\tau}\boldsymbol{\Sigma}\boldsymbol{\Theta}\boldsymbol{\Sigma}'(\mathrm{e}^{-\mathbf{K}\tau})'\, d\tau \tag{32}$$

where $\boldsymbol{\Theta}\, dt = (d\mathbf{w}_t)(d\mathbf{w}_t)'$. Thus

$$\mathrm{E}_t^Q(x_i(T)) = \begin{cases} x_1(t) - \lambda_1(T-t) & i = 1 \\ \mathrm{e}^{-\kappa_i(T-t)}x_i(t) - \dfrac{1 - \mathrm{e}^{-\kappa_i(T-t)}}{\kappa_i}\lambda_i & i = 2,\ldots,N \end{cases} \tag{33}$$

$$\mathrm{Cov}_t^Q(x_i(T), x_j(T)) = \begin{cases} \sigma_1^2(T-t) & i = 1, j = 1 \\ \sigma_i\sigma_j\rho_{ij}\dfrac{1 - e^{-(\kappa_i+\kappa_j)(T-t)}}{\kappa_i + \kappa_j} & i \neq 1, j \neq 1 \end{cases} \tag{34}$$

The valuation formula (17) is obtained by inserting Equations (33) and (34) into Equation (30).

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The valuation of multidimensional American real options using the LSM simulation method

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Abstract

In this paper we show how a multidimensional American real option may be solved using the LSM simulation method originally proposed by Longstaff and Schwartz [2001, The Review of the Financial Studies 14(1): 113–147] for valuing a financial option and how this method can be used in a complex setting. We extend a well-known natural resource real option model, initially solved using finite difference methods, to include a more realistic three-factor stochastic process for commodity prices, more in line with current research. Numerical results show that the procedure may be successfully used for multidimensional models, expanding the applicability of the real options approach.

Even though there has been an increasing literature on the benefits of using the contingent claim approach to value real assets, limitations on solving procedures and computing power have often forced academics and practitioners to simplify these real option models to a level in which they loose relevance for real-world decision making. Real option models present a higher challenge than their financial option counterparts because of two main reasons: First, many real options have a longer maturity which makes risk modeling critical and may force considering many risk factors, as opposed to the classic Black and Scholes approach with only one risk factor. Second, real investments many times exhibit a more complex set of interacting American options, which make them more difficult to value. In recent years new approaches for solving American options have been proposed which, coupled with an increasing availability of computing power, have been successfully applied to solving long-term financial options. In this paper we explore the applicability of one the most promising of these new methods in a multidimensional real option setting.

Keywords: Real options; Simulation; Natural resources; Valuation; Finance

1. Introduction

Even though in the last two decades there has been an increasing literature on the benefits of using the contingent claim approach to value real assets, limitations on solving procedures and computing power have often forced academics and practitioners to simplify these real option models to a level in which they loose relevance for real-world decision making.

There are two main reasons why real option models may present a higher challenge than their financial option counterparts to be solved. First, many real options have a longer maturity which makes risk modeling critical and may

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force the use of several risk factors, as opposed to only one, like in the classic Black and Scholes [1] stock-option model. Second, often real investments exhibit a more complex set of nested and interacting American options, which make them more difficult to value.

In the valuation of natural resource investments, for example, until only a few years ago most commodity price models considered only one risk factor and constant risk-adjusted returns. These earlier models have several undesirable implications, including that all futures returns should be perfectly correlated and exhibit the same volatility, which is not in line with empirical evidence. In recent years, however, many multifactor models of commodity prices have been proposed being much more successful than previous one-factor models in capturing the observed behavior of commodity prices like mean-reversion and a declining volatility term-structure [2,5–7].

On the other hand, the real options literature has also evolved and models increasingly take into account the different types of flexibilities available to decision makers when managing their projects. These flexibilities include the options to abandon a project, to shut down production, to delay investments, to expand capacity, to reduce costs through learning, among many others [8–11].

The introduction of multifactor price models into these real option models with many interacting flexibilities increases the difficulty of solving them, making traditional numerical approaches, like the finite difference methods, clearly inadequate. There has been, however, new research on using some sort of computer-based simulation procedures for solving American options, which coupled with an increasing availability of computing power, has been successfully applied to solving multifactor financial options. [12–18]. One of the most promising new approaches in this literature is the LSM method proposed by Longstaff and Schwartz [19] which has been tested for some financial options of limited complexity [20–22].

In this paper we explore the applicability of the LSM method in a multidimensional real option setting. We extend the Brennan and Schwartz [23] one-factor model for valuing a copper mine initially solved using finite difference methods, to include a more realistic three-factor stochastic process for commodity prices, more in line with current research. We implement the LSM method and discuss how complexity may be reduced. Numerical results show that the procedure may be successfully used for multidimensional models, notably expanding the applicability of the real options approach.

The remainder of this paper is organized as follows. Section 2 presents the problem to be solved. It describes the classic Brennan and Schwartz [23] real option model of a natural resource investment and how we extend it to include a multifactor model of commodity prices. A brief explanation on the real options approach for valuing investments is also included. Section 3 presents the proposed computer-based simulation procedure. Section 4 discusses the results of the numerical solution to the original and to the extended Brennan and Schwartz model and some implementation issues for high-dimensional models. Finally, Section 5 concludes.

2. The problem

2.1. The Real options approach to valuation

Real option valuation (ROV), can be understood as an adaptation of the theory of financial options to the valuation of investment projects. ROV recognizes that the business environment is dynamic and uncertain, and that value can be created by identifying and exercising managerial flexibility.

Options are contingent claims on the realization of a stochastic event, with ROV taking a “multi-path” view of the economy. Given the level of uncertainty, the optimal decision-path cannot be chosen at the outset. Instead, decisions must be made sequentially, hopefully with initial steps taken in the right direction, actively seeking learning opportunities, and being prepared to appropriately switch paths as events evolve.

ROV presents several improvements over traditional discount cash flow (DCF) techniques. First it includes a better assessment of the value of strategic investments and a better way of communicating the rationale behind that value. In most traditional DCF valuations, a base value is calculated. Then, this base value is “adjusted” heuristically to capture a variety of critical phenomena. Ultimately, the total estimated value may be dominated by the “adjustment” rather than the “base value.” With ROV, the entire value of the investment is rigorously captured. Conceptually, this includes the “base value” and the “option premium” obtained from actively managing the investment and appropriately exercising options.

Second, ROV provides an explicit roadmap or "optimal policy" for achieving the maximum value from a strategic investment. Most traditional investment valuations boil down to a number, and perhaps a set of assumptions underlying that number. However, the management actions required over time to realize that value are not clearly identified. With ROV, the value estimate is obtained specifically by considering these management actions. As a result, ROV indicates precisely which events are important and the necessary actions required to achieve maximum value.

There is a broad literature on ROV and how to maximize contingent claim value over all available decision strategies. Among them, Majd and Pindyck [24] include the effect of the learning curve by considering that accumulated production reduces unit costs, Trigeorgis [25] combines real options and their interactions with financial flexibility, McDonald and Siegel [26] and Majd and Pindyck [27] optimize the investment rate, and He and Pindyck [28] and Cortazar and Schwartz [29] consider two optimal control variables.

The ROV approach has been used to analyze uncertainty on many underlying assets, including exchange rates [30], costs [31] and commodities [32]. Real asset models have included natural resource investments, environmental, new technology adoption, and strategic options, among others [32–35].

Recently real options analysis is gradually advancing into the domain of strategic management and economic organization. Bernardo and Chowdry [11] analyze the way in which the organization learns from its investment projects. A related model is presented in [36]. They study the choice between a small and a large project, where choosing the small project allows one to re-invest later in the large project. Lambrecht and Perraudin [37] introduce incomplete information and preemption into an equilibrium model of firms facing real investment decisions. Miltersen and Schwartz [38] develop a model to analyze patent-protected R&D investment projects when there is imperfect competition in the development and marketing of the resulting product. Finally, Murto et al. [39] present a modeling framework for the analysis of investments in an oligopolic market for a homogenous commodity.

In this paper, we extend and solve the well-known Brennan and Schwartz [23] model for valuing natural resource investments. Other papers on natural resource investments include [40–45], among many others.

2.2. The Brennan and Schwartz [23] Model

The valuation of a copper mine in [23] laid the foundations for applying option pricing arbitrage arguments to the valuation of natural resource investments. In the model the value-maximizing policy under stochastic output prices considers the optimal timing of path-dependent, American-style options to initiate, temporarily cease or completely abandon production. We now describe the optimization problem in a general framework for valuing a switching option.

Consider the Brennan and Schwartz [23] model as a switching option with value $V_t(\mathbf{x}, j)$ and cash flows $CF_t(\mathbf{x}, j)$ at time t, which depend on a vector of N state variables, $\mathbf{x} = (x^1, \ldots, x^N)$ and the state of production j. The model considers that there are K states of production and the switching option can move from one state, j, to another, i, paying the corresponding switching cost, $C_t^{j,i}(\mathbf{x})$. This state switches can be made at any of $T+1$ stages, with $t = t_0, t_1, \ldots, t_T$.

We assume, for simplicity that the process for the state variables can be risk-adjusted and that markets are complete. Thus we can use the standard option pricing technique, which means that the switching option can be valued as the discounted expectation under the risk-neutral probability measure. At maturity, we assume the switching option has no value, thus:

$$V_T(\mathbf{x}, j) = 0; \quad j = 1, \ldots, K. \tag{1}$$

The switching option can then be solved recursively as follows. Moving backwards in time, in $t = T - \Delta t$ the value of the option is maximized among all feasible future stages:

$$V_{T-\Delta t}(\mathbf{x}, j) = \max_{i=1,\ldots,K} \left\{ CF_{T-\Delta t}(\mathbf{x}, i) - C^{j,i}_{T-\Delta t}(\mathbf{x}) \right\}; \quad j = 1, \ldots, K. \tag{2}$$

At times $t = t_0, t_1, \ldots, t_{T-2\Delta t}$ the value of the option can be computed as a function of current cash flows and the conditional expectation of the value in the following period. For example in $t_{T-2\Delta t}$:

$$V_{T-2\Delta t}(\mathbf{x}, j) = \max_{i=1,\ldots,K} \left\{ CF_{T-2\Delta t}(\mathbf{x}, i) + E_{T-2\Delta t}\left[V_{T-\Delta t}(\mathbf{x}, i) \right] \mathrm{e}^{-r\Delta t} - C^{j,i}_{T-2\Delta t}(\mathbf{x}) \right\}; \\ j = 1, \ldots, K, \tag{3}$$

where r is the risk free rate between time $t_{T-2\Delta t}$ and $t_{T-\Delta t}$. $E_{T-2\Delta t}$ [.] represents the conditional expectation at time $t_{T-2\Delta t}$ under the risk neutral probability measure. Consequently, the initial value of the switching option $V_0(\mathbf{x}, j^*)$ can be solved by this backward recursion where j^* represents the initial state.[1]

To determine the critical vector of state variables $\mathbf{x^c}$ that triggers the transition between different states of production, we must find the values that equate the conditional expectations between states of production.

In the original Brennan and Schwartz [23] the project is a contingent claim on copper price which follows a one-factor model, thus:

$$\frac{\mathrm{d}S_t}{S_t} = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}z, \tag{4}$$

in which μ is the instantaneous price return, σ is the return volatility and $\mathrm{d}z$ is an increment to a standard Gauss–Wiener process.

Commodity holders are assumed to receive, in addition to the price return, a convenience yield which does not accrue to the holder of a financial instrument contingent on copper, i.e. a futures contract. This convenience yield, C, is assumed to be proportional to the spot price, thus the risk-adjusted process for commodity prices may be written as:

$$\frac{\mathrm{d}S_t}{S_t} = (r - c)\,\mathrm{d}t + \sigma\,\mathrm{d}z \tag{5}$$

with r being the risk-free interest rate.

The initial amount of copper reserves is $Q_{\max}$, and the mine produces at a constant rate of q, so there are R feasible states of reserves, where

$$R = \frac{Q_{\max}}{q\Delta t}.$$

Also the mine may be open, closed or abandoned, so there are $3R$ states of production. The cost of switching between states depends on K_1, K_2 and M, with K_1 being the cost of closing an open mine, K_2 being the cost of opening a closed mine, and M the annual cost of maintaining a closed mine. The mine is abandoned at no cost when market value reaches zero. The unit cost of production is A, thus the cash-flow, when the mine is open, is

$$CF(S_t) = q(S_t - A) - \tau,$$

where τ includes annual income and royalty tax payments. In addition there is an annual property tax amounting to a fraction λ_1 or λ_0 of market value, depending on whether the mine is open or closed. When closed, the mine has no earnings, but incurs in a maintenance annual cost of M.

2.3. Extending the Brennan and Schwartz [23] Model

Initial applications of the real options approach were made in the natural resource sector mainly because of its high irreversible investments and the well developed commodity futures markets. Even though real option models, like the one we just described, have been successful in capturing many managerial flexibilities, in general they have considered very simple specifications of the price risk process, hindering the use of this approach in real-world applications.

This simple risk specification represented the state-of-the art in commodity price modeling when this approach was developed more than two decades ago. Since then much research has been done to capture in a better way the commodity price stochastic process, but real option models have not kept pace with this research, probably in part due to the added complexity to obtain numerical solutions in a multi-factor setting.

In this section we extend the Brennan and Schwartz [23] model to include a multifactor specification for uncertainty, model which in later sections will be solved numerically.

Commodity price processes differ on how convenience yield is modeled and on the number of factors used to describe uncertainty. Early models, i.e., Brennan and Schwartz [23], assumed a constant convenience yield and a one-factor Brownian motion. Later on, mean reversion in spot prices began to be included as a response to evidence that futures

[1] Later in the paper we add to this notation the subscript ω to indicate a simulated path.

return volatility declines with maturity. One-factor mean reverting models can be found, for example, in [46–48]. With one-factor models, however, all futures returns are assumed to be perfectly correlated which is not consistent with empirical evidence.

To account for a more realistic price behavior, two-factor models, with mean reversion, were introduced. Examples are [2–4]. Later, Cortazar and Schwartz [7] proposed a three-factor model for commodity prices and estimated it using oil futures, showing that the model exhibits low estimation errors.

In this paper we calibrate the Cortazar and Schwartz [7][2] three-factor model with copper futures and use it as an extension of the Brennan and Schwartz [23] model of a copper mine.

The model has three state variables, the commodity spot price, S_t, the demeaned convenience yield, y_t, and the expected long-term spot price return, v_t. Commodity spot prices follow a geometric Brownian motion. Spot price returns have an instantaneous drift equal to the expected long-term return, v_t, minus short-term deviations from the convenience yield, y_t. Both y_t and v_t are mean reverting, the first one to zero and the second one to a long-term average, $\bar{v}$.

The authors show that the three factors allow for an increased flexibility of the model which makes it able to match both the shape of the futures price curves and also the volatility term structure, two key attributes for price model selection.

The dynamics of the state variables are:

$$\frac{\mathrm{d}S_t}{S_t} = (v_t - y_t)\,\mathrm{d}t + \sigma_1\,\mathrm{d}z_1, \tag{6}$$

$$\mathrm{d}y_t = -\kappa y_t\,\mathrm{d}t + \sigma_2\,\mathrm{d}z_2, \tag{7}$$

$$\mathrm{d}v_t = a(\bar{v} - v_t)\,\mathrm{d}t + \sigma_3\,\mathrm{d}z_3, \tag{8}$$

with

$$\mathrm{d}z_1\,\mathrm{d}z_2 = \rho_{12}\,\mathrm{d}t, \quad \mathrm{d}z_1\,\mathrm{d}z_3 = \rho_{13}\,\mathrm{d}t, \quad \mathrm{d}z_2\,\mathrm{d}z_3 = \rho_{23}\,\mathrm{d}t. \tag{9}$$

Defining λ_i as the risk premium for each of the three risk factors, the risk-adjusted processes are:

$$\frac{\mathrm{d}S_t}{S_t} = (v_t - y_t - \lambda_1)\,\mathrm{d}t + \sigma_1\,\mathrm{d}z_1^*, \tag{10}$$

$$\mathrm{d}y_t = (-\kappa y_t - \lambda_2)\,\mathrm{d}t + \sigma_2\,\mathrm{d}z_2^*, \tag{11}$$

$$\mathrm{d}v_t = (a(\bar{v} - v_t) - \lambda_3)\,\mathrm{d}t + \sigma_3\,\mathrm{d}z_3^*, \tag{12}$$

with

$$(\mathrm{d}z_1^*)(\mathrm{d}z_2^*) = \rho_{12}\,\mathrm{d}t, \quad (\mathrm{d}z_2^*)(\mathrm{d}z_3^*) = \rho_{23}\,\mathrm{d}t, \quad (\mathrm{d}z_1^*)(\mathrm{d}z_3^*) = \rho_{13}\,\mathrm{d}t. \tag{13}$$

Following the same estimation procedure used in Cortazar and Schwartz [7] for oil prices, we calibrate this model for copper using all futures traded between 1991 and 1998 at NYMEX, obtaining the parameter values shown in Table 1.

The model allows for all three state variables to be correlated, providing a greater flexibility which is in line with empirical evidence. It is interesting to note that most parameter values, including the factor correlations, exhibit a sign and magnitude similar to those reported in [7] for oil. Also, the model fits the empirical data with a mean absolute error of 0.2% and exhibits similar theoretical and empirical volatilities, as shown in Fig. 1.

Using this three-factor price model to extend the Brennan and Schwartz [23] real option model we obtain a much better model specification. With this new price process, and following the general framework described in the previous section, we have that the switching option now depends on three state variables.

[2] Cortazar and Schwartz [7] is an extension of the Schwartz [3] model for commodity prices, and shares some of its good properties like mean reversion while ensuring positive prices. Other commodity price models could have been used, including square-root processes, stationary models or general affine models [49].

Table 1
Parameter values of the Cortazar and Schwartz [7] three-factor commodity price model calibrated using all copper futures traded between 1991 and 1998 at NYMEX

Parameters	Value
λ_1	−0.032
λ_2	−0.392
λ_3	−0.193
a	1.379
κ	2.850
$\bar{v}$	−0.007
σ_1	0.257
σ_2	0.906
σ_3	0.498
ρ_{12}	0.215
ρ_{23}	0.841
ρ_{13}	−0.229

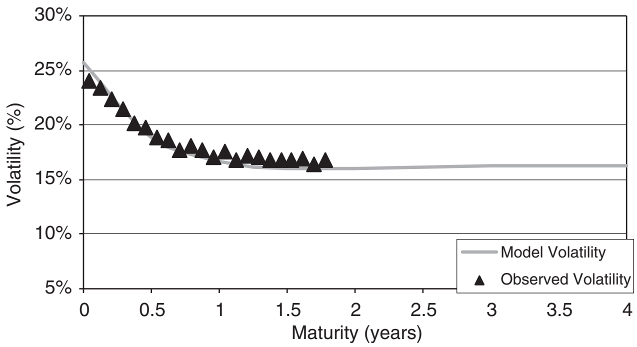

Fig. 1. Empirical and theoretical volatility term structure using the Cortazar and Schwartz [7] three-factor commodity price model calibrated using all copper futures traded between 1991 and 1998 at NYMEX.

Even though this model may be solved with traditional finite difference methods, is solved much more efficiently using the simulation method shown in the following sections.

3. Implementation

3.1. An introduction to the LSM method

We propose solving multidimensional problems, like the extended Brennan and Schwartz model, using the LSM method. To illustrate the LSM method proposed in Longstaff and Schwartz [19], we consider throughout this section a very simple copper mine that may extract all available resources instantaneously at any moment during the concession period. Also copper prices are considered in this section to follow a one-factor model. In the next section we will show how to implement the extended Brennan and Schwartz three-factor model.

Consider a simplified copper mine in which all reserves, Q, may be instantaneously extracted at any point in time incurring in a unit production cost of A. The copper spot-price, S_t, is assumed to follow a one-factor geometric Brownian motion:

$$\frac{\mathrm{d}S_t}{S_t} = (r - c)\,\mathrm{d}t + \sigma\,\mathrm{d}z \tag{14}$$

with r the risk-free interest rate and c the convenience yield.

The method starts by simulating a discretization of Eq. (14):

$$S_t = [1 + (r - c)\Delta t]\, S_{t-1} + S_{t-1}\sigma\sqrt{\Delta t}\,\varepsilon_t \tag{15}$$

with Δt the time interval in years and ε_t a random variable with a standard normal distribution.

Then, Eq. (15) is simulated through time, obtaining a price-path ω. The process is repeated N times, and a price matrix $\mathbf{S}$, with N price paths over a time horizon T, is obtained.

Like in any American option valuation procedure, the optimal exercise decision at any point in time is obtained as the maximum between the immediate exercise value and the expected continuation value. Given that the expected continuation value depends on future outcomes, the procedure must work its way backwards, starting from the end of the time horizon, T.

Starting with the last price in each path, ω, given that at expiration the expected continuation value is zero, the option value in T for the price path ω can be computed as

$$C(S_T(\omega)) = \text{Max}(Q(S_T(\omega) - A); 0). \tag{16}$$

One time-step backward, at $t = T - \Delta t$, the process is repeated for each price path, but now expected continuation value must be computed. It is important to notice that at this last time-step the expected continuation value may be computed using the analytic expression for a European option.

The main contribution of the LSM method is to compute the expected continuation value for all previous time-steps by regressing the discounted future option values on a linear combination of functional forms of current state variables. Given that the way these functional forms are chosen is not straightforward, in most of the paper we use simple powers of all state variables (monomials) and their cross products which is the most common implementation of the method found in the literature. In the last section of the paper we revisit this decision and provide alternative functional forms, which in our tests have shown to be computationally efficient in multidimensional settings.

In particular, let L^j, with $j = 1, 2, \ldots M$, be the basis of functional forms of the state variable $S_{T-\Delta t}(\omega)$ used as regressors to explain the realized present value in trajectory ω, then the least square regression is equivalent to solving the following optimization problem:

$$\underset{\{\mathbf{a}\}}{\text{Min}} \sum_{\omega=1}^{\text{N}} \left[C(S_T(\omega))\mathrm{e}^{-r\Delta t} - \sum_{j=1}^{M} a^j L^j(S_{T-\Delta t}(\omega)) \right]^2. \tag{17}$$

The optimal coefficients $\hat{\mathbf{a}}$ are then used to estimate the expected continuation value $\hat{G}(S_{T-\Delta t}(\omega))$:

$$\hat{G}(S_{T-\Delta t}(\omega)) = \sum_{j=1}^{M} \hat{a}^j L^j(S_{T-\Delta t}(\omega)). \tag{18}$$

Fig. 2 shows discounted continuation values of our simple copper mine for all N simulated paths and the expected continuation function computed as the solution to the regression of these values on powers of the spot copper price.

Then, the optimal decision for each price path is to choose the maximum between two values: the immediate exercise and the expected continuation value.

Once we have worked ourselves backwards until $t = 0$, we have a final vector of continuation values for each price-path, which averaged provides us with an estimation of its expected value, which in turn, when compared with the immediate exercise value gives the option value at time $t = 0$:

$$\text{Option value} = \text{Max}[Q(S_0 - A); \hat{G}(S_0)]. \tag{19}$$

3.2. Implementing the extended Brennan and Schwartz model

In this section we show how to implement the LSM approach to solve the Brennan and Schwartz [23] model for any price process, including the options to abandon a mine, to close an open mine and to open a closed mine.

Fig. 3 may be useful to understand the nature of the problem by describing all possible states during the simulation. It can be seen that as time evolves from 0 to T, the state variables that describe the three-factor dynamics for copper price,

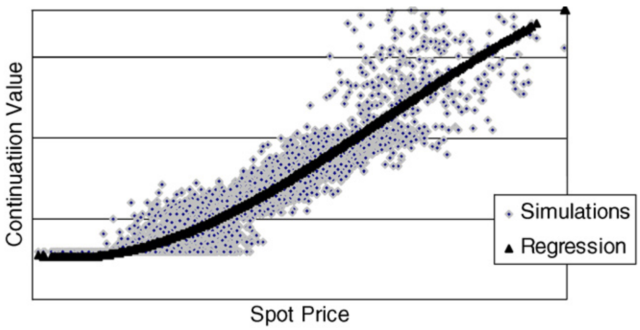

Fig. 2. Implementation of the LSM in the simple copper mine: discounted continuation values for all *N* simulated paths and expected continuation function computed from a regression on powers of the spot copper price.

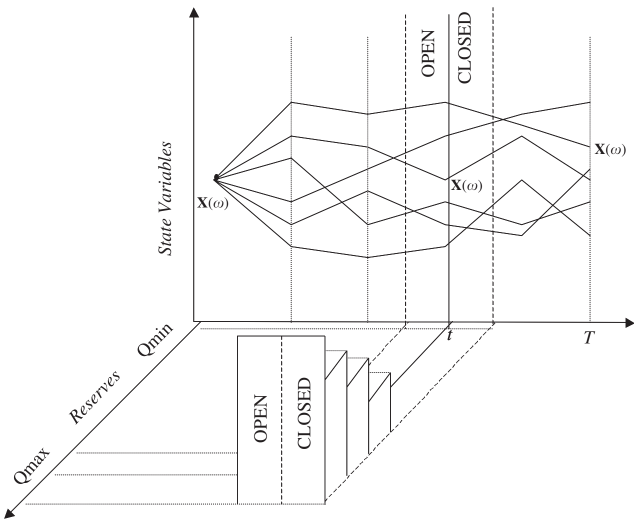

Fig. 3. State-space representation of the Brennan and Schwartz [23] model.

$\mathbf{x}(\omega)=[S(\omega),\, y(\omega),\, \upsilon(\omega)]$, evolve following different paths. At any point in time, and for any value of the three state variables, the mine may have any amount of copper reserves between zero and the initial reserves $Q_{\max}$. In addition, the mine at that point may be open or closed with market values $V_t(\mathbf{x}(\omega), Q)$ or $W_t(\mathbf{x}(\omega), Q)$,[3] respectively.

For each state of the system and for each operating policy, there is an associated cash flow for the mine. For example, when the mine is open and the operating policy is to remain open during Δt years producing q, the cash flow, *CF*, is

$$CF(S,q)=q\Delta t(S-A)-\tau. \tag{20}$$

Recall that for any price model, the spot price depends on the state variables $\mathbf{x}$, i.e. $S=f(\mathbf{x})$. In particular, for the three-factor Cortazar and Schwartz [7] model used in this paper, we have:

$$S=f(\mathbf{x})=\mathbf{h}'\mathbf{x} \quad \text{with} \quad \mathbf{h}'=[1\ \ 0\ \ 0]. \tag{21}$$

Also, as noted previously, the mine may be open, closed or abandoned, and may switch from one operating state to another incurring in fixed costs.

Fig. 4 summarizes the cash flows of an open mine which will either remain open, be closed or abandoned during time *t*. Fig. 5 shows the same information, but for a closed mine.

[3] In Section 2.2 the status of the mine (open or closed) was indicated using the variable *j*.

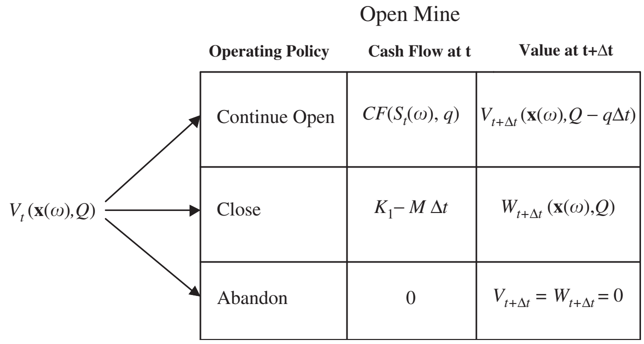

Fig. 4. Cash flows and value of an open mine as a function of the operating policy.

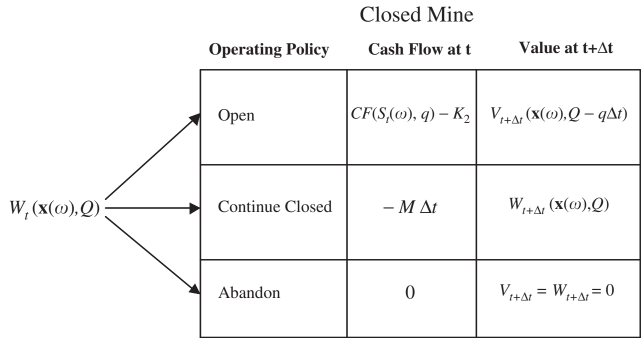

Fig. 5. Cash flows and value of a closed mine as a function of the operating policy.

As described earlier, after simulating all price paths from time zero to time T, the method requires making optimal decisions starting at time T and then working backwards until time zero is reached. The optimal decision at each point is taken by maximizing market value among all available alternatives.

At time T, given that the concession ends, the value of both the open and the closed mine is zero:

$$V_T(\mathbf{x}(\omega), Q) = W_T(\mathbf{x}(\omega), Q) = 0 \quad \forall Q,\ \forall \omega. \tag{22}$$

Then, at $t = T - \Delta t$ there is no time left to change the operating policy so there is no need to estimate an expected continuation value. So the market values are:

$$V_{T-\Delta t}(\mathbf{x}(\omega), Q) = \mathrm{Max}(CF(S_{T-\Delta t}(\omega), q); 0) \quad \forall Q, \tag{23}$$

$$W_{T-\Delta t}(\mathbf{x}(\omega), Q) = \mathrm{Max}(CF(S_{T-\Delta t}(\omega), q) - K_2; 0) \quad \forall Q. \tag{24}$$

Then, at $t = T - 2\Delta t$ we must estimate the expected continuation value. We regress the discounted mine value on a linear combination of functional forms of the state variables L(X), for each inventory level Q:

$$[V_{T-\Delta t}(\mathbf{X}, Q)\mathrm{e}^{-(r+\lambda_1)\Delta t} | W_{T-\Delta t}(\mathbf{X}, Q)\mathrm{e}^{-(r+\lambda_0)\Delta t}] = \mathbf{L}_{T-2\Delta t}(\mathbf{X})[a_{V,Q,T-2\Delta t} | a_{W,Q,T-2\Delta t}] + \mathrm{e}. \tag{25}$$

Once the optimal coefficients are found we can estimate the expected continuation values at $t = T - 2\Delta t$:

$$[\hat{\mathbf{G}}_{V,Q,T-2\Delta t} | \hat{\mathbf{G}}_{W,Q,T-2\Delta t}] = \mathbf{L}_{T-2\Delta t}(\mathbf{X})[\hat{a}_{V,Q,T-2\Delta t} | \hat{a}_{W,Q,T-2\Delta t}]. \tag{26}$$

Table 2
Expected and realized value of an open mine as a function of the operating policy

Expected value	Optimal decision	Realized value
$CF(S_t(\omega), q) + \hat{G}_{v,Q-q\Delta t,t}(\mathbf{x}(\omega))$	Continue open	$V_t(\mathbf{x}(\omega), Q) = CF(S_t(\omega), q) + V_{t+\Delta t}(\mathbf{x}(\omega), Q - q\Delta t)e^{-(r+\lambda_1)\Delta t}$
$-K_1 - M\Delta t + \hat{G}_{W,Q,t}(\mathbf{x}(\omega))$	Close	$V_t(\mathbf{x}(\omega), Q) = -K_1 - M\Delta t + W_{t+\Delta t}(\mathbf{x}(\omega), Q)e^{-(r+\lambda_0)\Delta t}$
0	Abandon	$V_t(\mathbf{x}(\omega), Q) = 0$

Table 3
Expected and realized value of a closed mine as a function of the operating policy

Expected value	Optimal decision	Realized value
$-K_2 + CF(S_t(\omega), q) + \hat{G}_{v,Q-q\Delta t,t}(\mathbf{x}(\omega))$	Open	$W_t(\mathbf{x}(\omega), Q) = -K_2 + CF(S_t(\omega), q) + V_{t+\Delta t}(\mathbf{x}(\omega), Q - q\Delta t)e^{-(r+\lambda_1)\Delta t}$
$-M\Delta t + \hat{G}_{W,Q,t}(\mathbf{x}(\omega))$	Continue closed	$W_t(\mathbf{x}(\omega), Q) = -M\Delta t + W_{t+\Delta t}(\mathbf{x}(\omega), Q)e^{-(r+\lambda_0)\Delta t}$
0	Abandon	$W_t(\mathbf{x}(\omega), Q) = 0$

Table 4
Open mine values as a function of the initial operation decision

Continue open	$V_0(\mathbf{x}, Q) = CF(S_0, q) + \hat{G}_{V,Q-q\Delta t,t=0}(\mathbf{x})$
Close	$V_0(\mathbf{x}, Q) = -K_1 - M\Delta t + \hat{G}_{W,Q,t=0}(\mathbf{x})$
Abandon	$V_0(\mathbf{x}, Q) = 0$

Thus, the expected continuation value at time $t = T - 2\Delta t$, as a function of the price state vector $\mathbf{x}$, may be computed. For example, the value of an open mine with Q units of resources, conditional on the state vector $\mathbf{x}$, would be

$$\hat{G}_{V,Q,T-2\Delta t}(\mathbf{x}) = \sum_{j=1}^{M} \hat{a}^j_{V,Q,T-2\Delta t} L^j_{T-2\Delta t}(\mathbf{x}). \tag{27}$$

Given that we can compute the expected continuation value, we are now able to obtain the optimal operating decisions by maximizing current cash flows plus the present value of expected continuation values. For example, when the mine is open there are three available operating alternatives: to continue open, to close down operations, or to abandon the mine. Adding current cash flows to discounted expected continuation values for each of the three alternatives, the decision maker may choose the best course of action.

Table 2 shows, for each of the three alternatives, the expected present value (at time t), the optimal decision should this expected present value be the maximum among the alternatives, and the final value at time t using actual realizations of the price simulation (instead of expected values to avoid biases due to the Jensen's inequality) at time $t + 1$. Table 3 shows the same information, but when the mine is initially closed.

This procedure is repeated from $t = T - 2\Delta t$ until $t = \Delta t$. At $t = \Delta t$ mine values are averaged over all price paths to provide an initial estimate of the expected continuation value for the mine:

$$\hat{G}_{V,Q-q\Delta t,t=0}(\mathbf{x}) = \frac{1}{s} \sum_{\omega=1}^{s} V_{\Delta t}(\mathbf{x}(\omega), Q - q\Delta t)\mathrm{e}^{-(r+\lambda_1)\Delta t}, \tag{28}$$

$$\hat{G}_{W,Q,t=0}(\mathbf{x}) = \frac{1}{s} \sum_{\omega=1}^{S} W_{\Delta t}(\mathbf{x}(\omega), Q)e^{-(r+\lambda_0)\Delta t}. \tag{29}$$

Tables 4 and 5 show the initial mine values depending on the initial status and operating policy of the mine.

Finally, to determine the optimal operating policy the method must find the critical state variables, $\mathbf{x}^c$, which equate expected present values for different operating decisions.

Table 5
Closed mine values as a function of the initial operation decision

Open	$W_0(\mathbf{x}, Q) = -K_2 + CF(S_0, q) + \hat{G}_{V,Q-q\Delta t,t=0}(\mathbf{x})$
Continue closed	$W_0(\mathbf{x}, Q) = -M\Delta t + \hat{G}_{W,Q,t=0}(\mathbf{x})$
Abandon	$W_0(\mathbf{x}, Q) = 0$

Table 6
Conditions to determine critical state variables $\mathbf{x}^c$ for switching mine operation

Open to Closed	$CF(\mathbf{x}^c, q) + \hat{G}_{V,Q-q\Delta t,t}(\mathbf{x}^c) = -K_1 - M\Delta t + \hat{G}_{W,Q,t}(\mathbf{x}^c)$
Closed to Open	$-M\Delta t + \hat{G}_{W,Q,t}(\mathbf{x}^c) = -K_2 + CF(\mathbf{x}^c, q) + \hat{G}_{V,Q-q\Delta t,t}(\mathbf{x}^c)$
Open to Abandon	$CF(\mathbf{x}^c, q) + \hat{G}_{V,Q-q\Delta t,t}(\mathbf{x}^c) = 0$
Closed to Abandon	$-M\Delta t + \hat{G}_{W,Q,t}(\mathbf{x}^c) = 0$

Table 7
Restrictions on initial state variables and parameters of the Cortazar and Schwartz [7] model to induce a one-factor price process similar to the Brennan and Schwartz [23] model

Cortazar–Schwartz model	Brennan–Schwartz model
y_o	λ_2/κ
υ_0	$\overline{v} - \lambda_3/a$
λ_1	$\upsilon_0 - y_0 - (r - c)$
λ_2	≈ 0
λ_3	≈ 0
a	1
κ	1
$\overline{v}$	≈ 0
σ_1	σ
σ_2	≈ 0
σ_3	≈ 0
ρ_{12}	≈ 0
ρ_{23}	≈ 0
ρ_{13}	≈ 0

Table 6 shows how to find the critical state variables to close an open mine, to open a closed mine, or to abandon from an open or from a closed mine.

4. Results

4.1. Results for the one-factor Brennan and Schwartz [23] model

In this section we validate our proposed approach by applying it to the one-factor Brennan and Schwartz [23] real options model and comparing the results to those originally reported using traditional finite difference methods.

A simple way of validating our approach is to see the one-factor price process as a particular case of the more general three-factor process. In this way by restricting some parameter values we can perform a better test on the algorithm by using the same computer program to solve both models.

Table 7 shows how the Cortazar and Schwartz [7] three factor model may be restricted to behave as the one-factor model used in Brennan and Schwartz [23]:

The simulation program computed 50 000 price paths, assuming a maximum extraction time of 50 years with three opportunities per year to switch between operating states. This is an approximation to the continuous-time Brennan and Schwartz model which assumes an infinite concession time and infinite opportunities per year to switch operating states.

Table 8
Open and closed mine value as a function of spot price

Spot price (US$lb.)	Mine value finite difference method reported in [23]		Mine value Simulation method	
	Open	Closed	Open	Closed
0.4	4.15	4.35	4.2	4.4
0.5	7.95	8.11	7.93	8.12
0.6	12.52	12.49	12.51	12.49
0.7	17.56	17.38	17.51	17.31
0.8	22.88	22.68	22.8	22.6
0.9	28.38	28.18	28.29	28.09
1.0	34.01	33.81	33.89	33.69

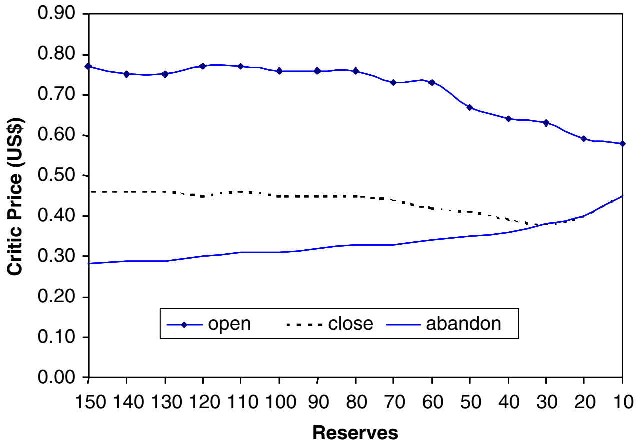

Fig. 6. Critical prices for opening, closing or abandoning a mine, as a function of reserve level obtained using the LSM method.

Table 8 compares the finite difference values reported in [23] with those obtained using the above simulation procedure. The mine and market parameters used are those reported in [23]. It can be seen that the simulation method converges to the known finite difference solution.

Our simulation procedure may also provide the optimal operating policy. Fig. 6 shows the critical prices for abandoning, opening a closed mine, and closing an open mine, as a function of reserves. Results are very similar to those reported in [23].

4.2. Results for the three-factor extension of the Brennan and Schwartz [23] model

We now report the solution to the Brennan and Schwartz [23] model extended to include the Cortazar and Schwartz [7] three-factor commodity price model. The parameter values used are those reported in Table 1.

We now assume a 30 year concession horizon, and three opportunities to switch operation states per year. To value the mine for a particular date, say April the 14th, 1999, we must first determine the values of the state variables S_o, y_o, υ_o corresponding to that date, which are 0.64, 0.198 and 0.244, respectively. Following the implementation procedure described in Section 4.1 we obtain a value for the open mine of MMUS$ 15.64, and for the closed mine of MMUS$ 15.52.

To explore how mine value changes according to variations in price conditions, we solve for the value of the mine for a 5 year time span. Results are reported in Fig. 7.

It is interesting to note that mine value exhibits mean reversion. Even though it is well known that copper prices do exhibit mean reversion, which is captured in the three-factor model, given that a mine produces copper during a long

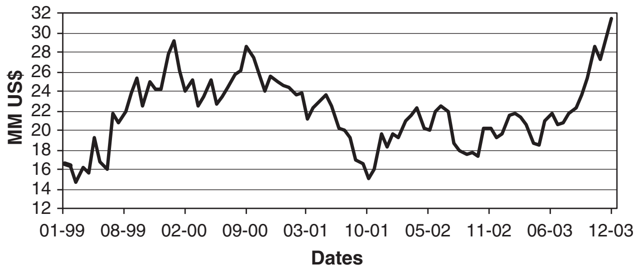

Fig. 7. Monthly values of the extended Brennan and Schwartz [23] open mine according to historical copper pricing conditions from January 1999–December 2003.

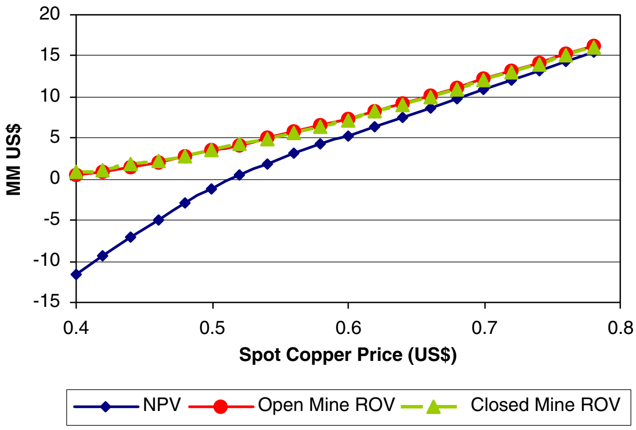

Fig. 8. Value of the open mine using ROV and NPV as a function of spot price for $y = 0.01$ and $v = -0.1$.

time horizon it could be thought that current spot prices would not have a great effect on mine values. Fig. 7 shows this is not the case.

Doing comparative static analysis on how mine value changes with variations in the spot price or in any individual state variable or parameter value is rather straightforward. For example, Fig. 8 shows how mine value increases with copper spot prices. It is also interesting to note how mine values are convex, because as mine value approaches zero the probability of abandoning the mine increases. Finally, the same figure compares mine value computed with the real option model to a simple net present value calculation which does not recognize operating flexibilities to abandon or close operations. It can be seen that when spot prices are lower, option values are greater and these two valuation methodologies diverge the most. By the same token, when prices are high, flexibilities are not too valuable and both valuations converge.

Comparative static analysis for the value or for the optimal policy can easily be performed for any of the state variables, strengthening the ability of the LSM method to study the behavior of an investment project for different scenarios.

4.3. An alternative implementation for multi-dimensional settings

In the previous sections we have shown a simple implementation of the LSM approach for solving a real options model with a three-factor price process. As stated previously, one of the main contributions of this approach is the computation of the expected continuation value by regressing discounted future option values on a linear combination of functional forms of current state variables. The way these functional forms are chosen is not straightforward and, as is discussed in this section, it may become an important issue in high-dimensional settings.

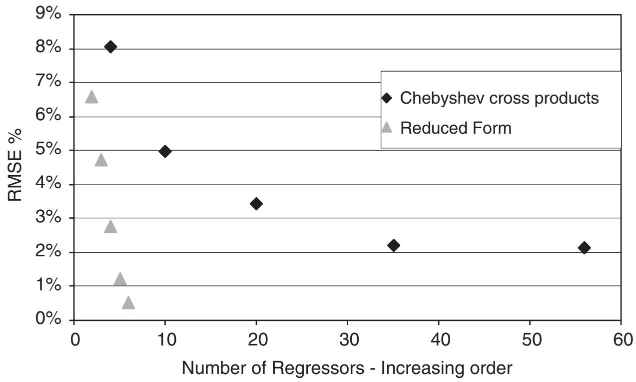

Fig. 9. RMSE as a function of the number of regressors for Chebyshev Polynomials and for the reduced-base form using only futures.

Longstaff and Schwartz [19] propose for multidimensional implementations of their method the use of basic functions from Laguerre, Chebyshev, Gegenbauer, Jacobi polynomials, or, the simple powers and cross products of the state variables used in this paper. For example, if the state variables were only two, X and Y, a simple order-two expected continuation value function would have six regressors, namely:

$$\hat{G}(X, Y) = \hat{a}_0 + \hat{a}_1 X + \hat{a}_2 Y + \hat{a}_3 XY + \hat{a}_4 X^2 + \hat{a}_5 Y^2. \tag{30}$$

Although this procedure for specifying the regression basis has the benefit of being simple and theoretically convergent [22,52,53], in high-dimensional settings it may induce numerical problems due to the least squares regression instability [21] and performance problems due to the high number of regressors.

An alternative to the described procedure for specifying the base that we have tested is to take advantage of the structure of the problem to be solved. Thus, given that optimal exercise of options depends on expected spot prices and volatilities, instead of using as regressors powers of all state variables, it could be better to use functions on futures, European options or bond prices, which have economic meaning.

Recent independent work has shown the potential of this approach for implementing multidimensional financial derivatives. For example Andersen and Broadie [50] include as regressors European call options and their powers for valuing a multi-stock option and Longstaff [51] value the prepayment option on a term structure string model with 120 state variables using closed form par-price bonds and their powers. We are not aware, however, of any use of a similar approach in the real options literature.

Thus our alternative implementation, in its simplest specification, boils down to computing the expected continuation value function:

$$\hat{G}_N(\mathbf{x}) = \hat{a}_0 + \sum_{i=1}^{N} \hat{a}_i E(S)^i, \tag{31}$$

where $E(S)$ is the expected spot price under the risk-adjusted measure, i.e., the future price.

Our tests show that using this reduced-base specification we can obtain similar valuation accuracy in a simpler way than using polynomials of state variables. For example, we solved a three-factor European option with known analytic solution with two alternative implementations of the LSM approach: Chebyshev functions and futures prices. Fig. 9 computes the RMSE as a function of the number of regressors, showing that using futures requires less regressors for any giver error level.

Using less regressors for estimating the continuation function has many computational benefits including reducing CPU-processing time which could be critical for high-dimensional implementations.

For example we performed another test solving the extended three-factor price model Brennan and Schwartz mine, obtaining valuations within 1% for both LSM implementations, while calculation time increased with the number of regressors, as shown in Fig. 10. These results suggest that if calculation time is an issue it is worth exploring alternative implementations of the LSM approach.

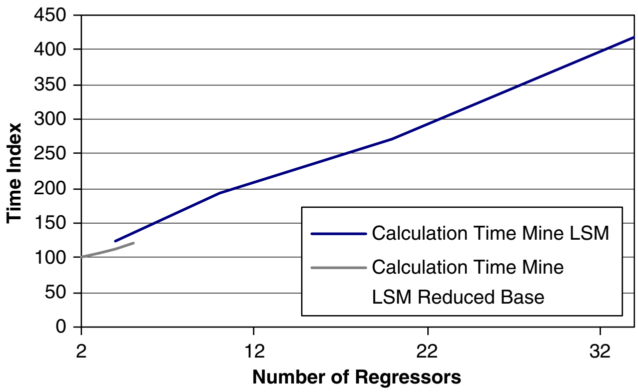

Fig. 10. Relative computer calculation time for solving the extended Brennan and Schwartz mine model as a function of the number of regressors when using the standard and the reduced base implementation of the LSM method.

5. Conclusions

Real options valuation (ROV) is an emerging paradigm that provides helpful insights for both valuing and managing real assets. It provides more precise quantifications on the value of available strategic and operational flexibilities than traditional discounted cash flow techniques.

Despite its potential, the ROV approach has not yet made a strong inroad in corporate decision-making due to several reasons, one of which is the requirement to keep models too simple to obtain solutions within a reasonable amount of effort.

In this paper we show how it is possible to solve complex multidimensional American options using computer-based simulation procedures. The implementation is validated using the one-factor Brennan and Schwartz [23] model with the reported finite difference solution.

We then extend the Brennan and Schwartz [23] to include a three-factor price model and solve it using the proposed methodology. Comparative static analyses are provided.

This paper argues that these new simulation methods have the potential of expanding significantly the use of the ROV approach without having to compromise rigorous modeling in order to obtain a solution.

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